# Section 3.2
# Definition of the Derivative
# 2 Lectures

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MATHS 101: Calculus I

# Nondifferentiable function with a corner

## Example

Show that the function

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

is not differentiable at $x = 0$.

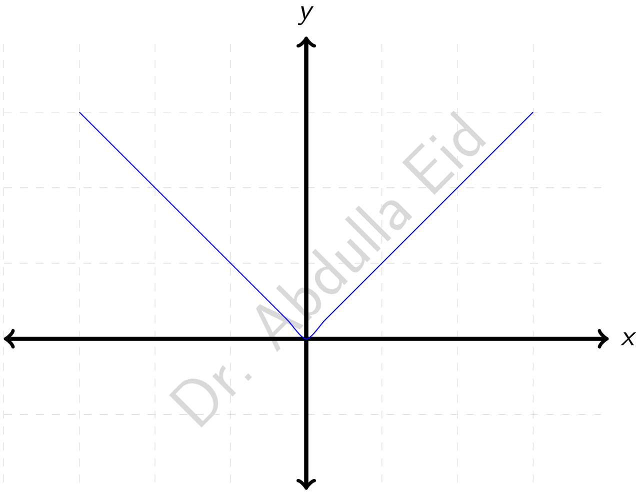
Solution: We will try to find the derivative of the function at $x = 0$,

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$f'(0) = \lim_{h \to 0} \frac{|0+h| - |0|}{h}$$

$$f'(0) = \lim_{h \to 0} \frac{|h|}{h} \text{ which is does not exist by Example ??}$$

So the function $f(x) = |x|$ is not differentiable at $x = 0$ (since we have a corner ! at $x = 0$).


y
x

# Nondifferentiable function with a cusp

## Example

Show that the function $f(x) = \sqrt{|x|}$ is not differentiable at $x = 0$.

Solution: We will try to find the derivative of the function at $x = 0$,

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$f'(0) = \lim_{h\to 0} \frac{\sqrt{|h|} - 0}{h}$$

We need to find the left and the right limit.

$$f'(0) = \lim_{h\to 0} \frac{\sqrt{|h|} - 0}{h}$$

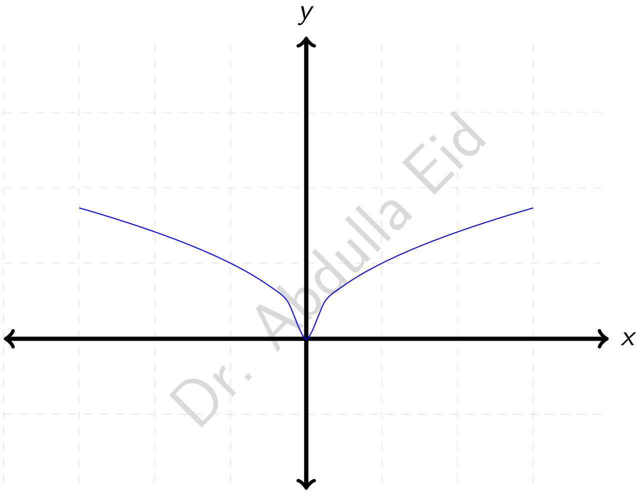
$$\begin{aligned}\lim_{h\to 0^+} \frac{\sqrt{|h|} - 0}{h} &= \lim_{h\to 0^+} \frac{\sqrt{h} - 0}{h} \\ &= \lim_{h\to 0^+} \frac{1}{\sqrt{h}} \\ &= \infty\end{aligned}$$

$$\begin{aligned}\lim_{h\to 0^-} \frac{\sqrt{|h|} - 0}{h} &= \lim_{h\to 0^-} \frac{\sqrt{-h} - 0}{h} \\ &= \lim_{h\to 0^-} \frac{1}{\sqrt{-h}} \\ &= \infty\end{aligned}$$

so we have

$$f'(0) = \infty$$

i.e., the slope is infinity so the function $f(x) = \sqrt{|x|}$ is not differentiable at $x = 0$ (since we have a cusp ! at $x = 0$).


y
x

# Nondifferentiable everywhere function



**Example**

Look at the function that results from everyday trading in stock, or FX, you will see that at every point we have either a cusp or corner. Example, look at www.finance.yahoo.com

# Differentiablity $\rightarrow$ Continuity

## Theorem

*If $f$ is differentiable at $x = a$, then $f$ is continuous at $x = a$.*

Solution: Assume $f$ is differentiable at $x = a$, i.e., the derivative $f'(a)$ (as a limit exist). We want to show that $f$ is continuous at $x = a$,i.e., $\lim_{x \to a} f(x) = f(a)$.

$$f(x) - f(a) = \frac{f(x) - f(a)}{(x - a)} \cdot (x - a)$$

$$\lim_{x \to a} f(x) - f(a) = \lim_{x \to a} \frac{f(x) - f(a)}{(x - a)} \cdot (x - a)$$

$$\lim_{x \to a} f(x) - f(a) = f'(a) \cdot 0 = 0$$

$$\lim_{x \to a} f(x) = f(a)$$

Therefore, the function is continuous at $x = a$.