Section 3.10 Related Rates 2 Lecture

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MATHS 101: Calculus I

Rate of change

Recall:

- If a line has a slope m = 3, it means that for every one step to the right, we move along the line 3 steps up. In this case, as x increases, y increases.
- If a line has a slope m = -2, it means that for every 1 step to the right, we move along the line 2 steps down. In this case, as x increases, y decreases.

For general function y = f(x), for every step to the right, how many steps to go up/down? How do we measure that change in y? If x changes by 1, an estimate of the change in y is $\frac{dy}{dx}$.

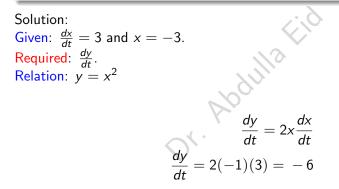
Definition

The derivative of y = f(x) can be interpreted as *rate of change* of y in term of x.

In this section, we have our variables are functions in t. so for example, we have

- $\frac{dx}{dt}$ is the rate change in x with respect to the time t. If $\frac{dx}{dt} = 4$, it means that for every one unit in time, x increases by 4 units.
- ⁽²⁾ $\frac{dr}{dt}$ is the rate change in r with respect to the time t. If $\frac{dr}{dt} = -2$ cm/second, it means that for every one second, r decreases 2 cm.

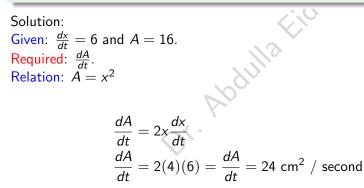
If
$$y = x^2$$
 and $\frac{dx}{dt} = 3$. What is $\frac{dy}{dt}$ when $x = -1$?



Air is being pumped in a spherical balloon so that its volume increases at a rate of 500 cm³/second. How fast is the radius of the balloon increases when the diameter is 50 cm? (Note that the volume of the sphere is $V = \frac{4}{3}\pi r^3$.

Solution:
Given:
$$\frac{dV}{dt} = 500$$
 and $D = 50 \rightarrow rr = \frac{D}{2} = 25$.
Required: $\frac{dr}{dt}$.
Relation: $V = \frac{4}{3}\pi r^3$
 $\frac{dV}{dt} = \frac{4}{3}\pi (3r^2)\frac{dr}{dt}$
 $500 = 4\pi (25)^2\frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{500}{4\pi (25)^2}$ cm / second

Each side of a square is increasing at a rate of 6 cm/second. At what rate is the area of the square increases when the area of the square is 16 cm^2 .



Exercise

The length ℓ of a rectangle is decreasing at the rate of 2 cm/second while the width w is increasing at the rate of 2 cm/second when $\ell = 12$ cm and w = 5 cm. Find the rate of change of

- Area
- 2 Perimeter
- Isength of the diagonal



A ladder is 5 m long rests against a vertical wall. If the bottom of the ladder moves away at rate 0.5 m/second. How fast the top of the ladder slides down when the bottom of the ladder is 4 m from the wall.

Solution:
Given:
$$\frac{dx}{dt} = 0.5$$
 and $\ell = 5$, $x = 4$.
Required: $\frac{dy}{dt}$.
Relation: $x^2 + y^2 = 25$
 $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2(4)(0.5) + 2(3)(\frac{dy}{dt}) = 0$
 $4 + 6\frac{dy}{dt} = 0 \rightarrow \frac{dy}{dt} = \frac{-4}{6}$ m / second

An airplane is moving parallel to the ground on elevation 10 km with a rate of 6 km/minute. Find the rate of change for the angle where an observer observes the airplane when his angle is $\frac{\pi}{3}$.

Solution:
Given:
$$\frac{dx}{dt} = 6$$
 and $y = 10$, $\theta = \pi/3$.
Required: $\frac{d\theta}{dt}$.
Relation: $\tan \theta = \frac{10}{x}$ (hence $x = \frac{10}{\tan(\frac{\pi}{3})}$)
 $\sec^2 \theta \cdot \frac{d\theta}{dt} = \frac{-10}{x^2} \cdot \frac{dx}{dt}$
 $2\frac{d\theta}{dt} = \frac{-10}{()^2} \cdot 6 \rightarrow \frac{d\theta}{dt} =$ radian / minute

A cube's surface area increases at a rate of 72 in²/second. At what rate the cube volume changes when the edge of the cube is 3 in? (Note that the volume of the cube is $V = x^3$ and the surface area is $S = 6x^2$)

