Section 5.4
Initial value problems
1 Lecture

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MATHS 104: Mathematics for Business II

Differential equations

Definition

The *Initial value problem* is the problem of finding the function y given the derivative y' and *initial condition* $y(a) = b$.

Idea: Integrate y' and find the general antiderivative and then substitute $x = a$ and $y = b$ to find the function y.

Note: The initial value problem is subclass of a bigger problem in mathematics called solving differential equations.

Example

(Old Exam Question) If y satisfies the given condition, find $y(x)$.

$$\frac{dy}{dx} = 6x^2 - 8x + 12 \text{ and } y(0) = 2$$

Solution: We integrate to find y.

$$\begin{aligned}
y &= \int (6x^2 - 8x + 12)\, dx \\
y &= 2x^3 - 4x^2 + 12x + C \\
2 &= y(0) \\
2 &= 2(0)^3 - 4(0)^2 + 12(0) + C \\
2 &= C \\
y &= 2x^3 - 4x^2 + 12x + 2
\end{aligned}$$

Exercise

(Old Exam Question) If y satisfies the given condition, find $y(x)$.

$$\frac{dy}{dx} = 9x^2 - 4x + 5 \text{ and } y(-1) = 0.$$

Example

If y satisfies the given condition, find $y(x)$.

$$y'' = 6x + 4 \text{ and } y'(0) = 1, y(0) = 5$$

Solution: We integrate to find y.

$$y' = \int (6x + 4)\, dx$$

$$y' = 3x^2 + 4x + C$$

$$1 = y'(0)$$

$$1 = 3(0)^2 + 4(0) + C \rightarrow C = 1$$

$$y' = 3x^2 + 4x + 1$$

$$y = \int (3x^2 + 4x + 1)\, dx$$

$$y = x^3 + 2x^2 + x + D$$

$$5 = y(0) \rightarrow D = 5$$

$$y = x^3 + 2x^2 + x + 5$$

Example

(Old Exam Question) If y satisfies the given condition, find $y(x)$.

$$\frac{dy}{dx} = \cos t + \sin t \text{ and } y(\pi) = 1$$

Solution: We integrate to find y.

$$
\begin{aligned}
y &= \int (\cos t + \sin t)\, dt \\
y &= \sin t - \cos t + C \\
1 &= y(\pi) \\
1 &= \sin \pi - \cos \pi + C \\
1 &= -1 + C \\
2 &= C \\
y &= \sin t - \cos t + 2
\end{aligned}
$$

Example

(Old Exam Question) If y satisfies the given condition, find $y(x)$.

$$\frac{dy}{dx} = e^{-x} + \frac{1}{\sqrt{1-x^2}} \text{ and } y(0) = \pi$$

Solution: We integrate to find y.

$$y = \int (e^{-x} + \frac{1}{\sqrt{1-x^2}})\,dt$$

$$y = -e^{-x} + \sin^{-1} x + C$$

$$\pi = y(0)$$

$$\pi = -e^{-0} + \sin^{-1}(1) + C$$

$$\pi = -1 + \frac{\pi}{2} + C$$

$$\frac{\pi}{2} - 1 = C$$

$$y = -e^{-x} + \sin^{-1} x + \frac{\pi}{2} - 1$$