University of Bahrain
Department of Mathematics
MATHS253: Set Theory
Fall 2018
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Homework 10: Proofs involving sets II

Do not turn it in, Solution will be presented in the class by you.

Name: _____

1. Prove the following statements about the sets

   1. In class we have proved that $P(A) \cup P(B) \subset P(A \cup B)$. Show that the equality does not hold.

   2. What is $|A \times B|$? (Prove your answer)

3. Let $A = \{1, 2\}$ and $B = \{2, 4\}$. Find the following
   (a) $A \times B$.

   (b) $P(A \times B)$.

   (c) $|P(A \times B)|$ and $|P(A)| \times |P(B)|$ and compare your answer.

4. Give a counter example to show that $(A \times B)^c = A^c \times B^c$.

5. Prove that $(A \times B) \cup (C \times D) \subset (A \cup C) \times (B \cup D)$.

6. Prove that $(A \cap B) \times C = (A \times C) \cap (B \times C)$.

7. consider the intervals $A = [2, 5]$ and $B = (0, 4)$.
   (a) What is $(A - B)^c$ in interval notation?

   (b) Draw a picture of $(A - B)^c \times (B - A)$.

   (c) For sets A and B, find a necessary and sufficient conditions for $A \times B = B \times A$. I.e., complete the proposition

   $$A \times B = B \times A \text{ if and only if } \cdots\cdots$$

8. For each n, define $B_n = \{n\} \times \mathbb{R}$.
   (a) Draw a picture of $B_1 \cup B_2 \cup B_3$

   (b) Draw a picture of $C = [0, 4] \times \{-2, 2\}$.

   (c) Compute $(B_1 \cup B_2 \cup B_2) \cap C$.

   (d) Compute $(B_1 \cap C) \cup (B_2 \cap C) \cup (B_3 \cap C)$.

   (e) Compare the two answers above. What do you notice?