University of Bahrain
Department of Mathematics
MATHS253: Set Theory
Fall 201
Dr. Abdulla Eid

Homework 12: Functions, part 1
Due December 20, 2018

Name: _____

1. Determine the domain, codomain, and the range for each of the following functions. Also mention which of these are injective, surjective, or bijective.

   (a) $f : \{a, b, c, d\} \to \{a, b, c, d\}$ by $f(a) = b, f(b) = a, f(c) = c, f(d) = d$.

   (b) $f : \{a, b, c, d\} \to \{a, b, c, d\}$ by $f(a) = b, f(b) = b, f(c) = d, f(d) = c$.

   (c) $f : \{a, b, c, d\} \to \{a, b, c, d\}$ by $f(a) = d, f(b) = b, f(c) = c, f(d) = d$.

   (d) $f : \mathbb{Z} \to \mathbb{Z}$ by $f(x) = x - 1$.

   (e) $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^3 - 1$.

2. What does it mean for a function to be **not** injective? surjective?

3. Determine whether the following functions are injective, surjective, or bijective. Prove your assertion.

   (a)

$$\begin{aligned} f : \mathbb{R} - \{1\} &\to \mathbb{R} \\ x &\mapsto \frac{5}{1-x} \end{aligned}$$

   (b)

$$\begin{aligned} f : \mathcal{P}(A) &\to \mathcal{P}(A) \\ X &\mapsto A - X \end{aligned}$$

4. (a) Find a set A such that the function $f : A \ni x \mapsto \sin x \in \mathbb{R}$ is injective.

   (b) Find a set B such that the function $f : \mathbb{R} \ni x \mapsto \sin x \in B$ is surjective.

5. Let $f : (-\infty, 0] \to \mathbb{R}$ and $g : [0, \infty) \to \mathbb{R}$ defined by

$$f(x) = x^2 \,, \qquad g(x) = \begin{cases} \dfrac{x}{1-x} \,, & x < 1 \\ \\ 1 - x \,, & x \geq 1 \end{cases}$$

   Does $g \circ f$ maps $(-\infty, 0]$ onto $\mathbb{R}$?

6. Let $f : A \to B$ and $g : B \to C$ be two functions.

   (a) Prove that if f, g are surjective functions, then $g \circ f$ is surjective.

   (b) What is the converse of the proposition in Part (a)? Is it true?

   (c) Prove that if $g \circ f$ is injective, then f is injective.