University of Bahrain
Department of Mathematics
MATHS253: Set Theory
Fall 2018
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Homework 13: Functions, part 2
Do not hand it in

Name: ______________________________

1. Let $f : A \to B$ be a bijective function. Show that $f^{-1} : B \to A$ is also a bijective function.

2. Define the function

$$
\begin{aligned}
f : \mathbb{R} - \{1\} &\to \mathbb{R} - \{1\} \\
x &\mapsto \frac{x}{x-1}
\end{aligned}
$$

(a) Prove that f is a bijective function.

(b) Find f^{-1}.

(c) Find $f \circ f \circ f$.

3. Let $f : A \to B$ and $g, h : B \to C$. Prove that if f is a bijective function and $g \circ f = h \circ f$, then $g = h$. Can you conclude that if f is a bijective, then it has a *unique* inverse?

4. Prove that the *identity* function on any set A is a bijective. What is the inverse of it?

5. Let $\mathcal{A} = \{A_i \,|\, i \in I\}$ be a collection of subsets of a set A and $f : A \to B$ be a function. Prove the following:

   (a) $f^{-1}\left(\bigcap_{i\in I} A_i\right) = \bigcap_{i\in I} f^{-1}(A_i)$

   (b) $f^{-1}\left(\bigcup_{i\in I} A_i\right) = \bigcup_{i\in I} f^{-1}(A_i)$

(c) $f\left(\bigcap_{i\in I} A_i\right) \subset \bigcap_{i\in I} f(A_i)$. Give an example to show that the equality does not hold. Moreover, prove that equality holds if f is injective.

(d) $f\left(\bigcup_{i\in I} A_i\right) = \bigcup_{i\in I} f(A_i)$. Give an example to show that the equality does not hold.