University of Bahrain
Department of Mathematics
MATHS253: Set Theory
Fall 2018
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Homework 3: Predicates and Quantifiers

Name: _____

1. Let $Q(x, y)$ denote the predicate "x is the capital of y". What are the truth values of the following:
   1. Q(Manama, Bahrain).
   2. Q(Toronto, Candana).
   3. Q(Dubai, UAE).
   4. Q(New York, USA).

2. Let $P(x)$ denote the predicate "x is tall", where x is a basketball player. Express each of the following proposition as an English sentences.
   (a) $\forall x, P(x)$.
   (b) $\exists x, P(x)$.
   (c) $\forall x, \neg P(x)$.
   (d) $\exists x, \neg P(x)$.

3. Let $P(x)$ be the statement $x = x^2$, where x is an integer. What are the truth values of the following (Why?):

   (a) $P(0)$. (b) $P(1)$. (c) $P(2)$.

   (a) $P(-1)$. (b) $\exists x, P(x)$. (c) $\forall x, P(x)$.

4. Prove or disprove: There exist integers m and n such that $2m - 3n = 15$.

5. State the negations of the following quantified statements:

   1. "For every rational number r, the number $\frac{1}{r} is rational$".

   2. "There exists a rational number r such that $r^2 = 2$".

   3. "For every two real numbers x and y, $x^2 + y^2 \geq 0$".

4. "There exists natural number a, such that for all natural numbers b, $ab = 1$".

5. "For all integers a and b, if ab is even, then a is even and b is even".
(Hint: Use De'Morgan laws and the fact that $\neg(p \to q) \equiv p \wedge \neg q$)

6. "There exist an integer x, such that x is odd and x^2 is even".

7. "If n is even, then n^2 is odd".

8. "x is an odd whenever $x + 1$ is even".

6. The notation $\exists! x, P(x)$ denotes the proposition

"There exists a *unique* x such that $P(x)$ is true".

What are the truth values of the following given x is an integer.

(a) $\exists! x, x > 1$.

(b) $\exists! x, x^2 = 1$.

(c) $\exists! x, x + 3 = 2x$.

(d) $\exists! x, x = x + 1$.

(e)* Express the proposition $\exists! x, P(x)$ using the quantifiers $\exists, \forall$ and logical operators only.