Section 12.7
Higher Derivative

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MATHS 104: Mathematics for Business II

Higher Derivative

Let $y = f(x)$

First Derivative	y'	$\frac{dy}{dx}$	$f'(x)$	$\frac{d}{dx}(f(x))$
Second Derivative	y''	$\frac{d^2y}{dx^2}$	$f''(x)$	$\frac{d^2}{dx^2}(f(x))$
Third Derivative	y'''	$\frac{d^3y}{dx^3}$	$f'''(x)$	$\frac{d^3}{dx^3}(f(x))$
Fourth Derivative	$y^{(4)}$	$\frac{d^4y}{dx^4}$	$f^{(4)}(x)$	$\frac{d^4}{dx^4}(f(x))$
nth Derivative	$y^{(n)}$	$\frac{d^ny}{dx^n}$	$f^{(n)}(x)$	$\frac{d^n}{dx^n}(f(x))$

Example

(Old Final Exam Question) Find $\frac{d^4y}{dx^4}$ for

$$y = 5 \ln x$$

Solution:

$$y' = 5\frac{1}{x}$$
$$= 5x^{-1}$$
$$y'' = -5x^{-2}$$
$$y''' = 10x^{-3}$$
$$y^{(4)} = -30x^{-4}$$
$$= \frac{-30}{x^4}$$

Example

(Old Final Exam Question) Find $\frac{d^3y}{dx^3}$ for

$$y = \frac{1}{1+2x}$$

Solution:

$$
\begin{aligned}
y &= \frac{1}{1+2x} \\
&= (1+2x)^{-1} \\
y' &= -2(1+2x)^{-2} \\
y'' &= 8(1+2x)^{-3} \\
y''' &= -48(1+2x)^{-4} \\
&= \frac{-48}{(1+2x)^4}
\end{aligned}
$$

Example

(Old Exam Question) Find $\frac{d^3y}{dx^3}$ for

$$y = x^4 e^x$$

Solution:

$$y' = 4x^3 e^x + x^4 e^x$$
$$y'' = (12x^2 e^x + 4x^3 e^x) + (4x^3 e^x + x^4 e^x)$$
$$= 12x^2 e^x + 8x^3 e^x + x^4 e^x$$
$$y''' = (24x e^x + 12x^2 e^x) + (24x^2 e^x + 8x^3 e^x) + (4x^3 e^x + x^4 e^x)$$
$$= 24x e^x + 36x^2 e^x + 12x^3 e^x + x^4 e^x$$

Example

(Old Exam Question) Find $\frac{d^3y}{dx^3}$ for

$$y = e^{x^3} + \sqrt[3]{333}$$

Solution:

$$y' = 3x^2 e^{x^3}$$
$$y'' = 6x e^{x^3} + 3x^2 e^{x^3} 3x^2$$
$$= 6x e^{x^3} + 9x^4 e^{x^3}$$
$$y''' = (6e^{x^3} + 6x e^{x^3}) + (36x^3 e^{x^3} + 9x^4 e^{x^3} 4x^2)$$
$$= 6e^{x^3} + 6x e^{x^3} + 36x^3 e^{x^3} + 36x^6 e^{x^3}$$

Example

(Old Final Exam Question) Find $\frac{d^2 y}{dx^2}$ for

$$y = x^3 \ln x$$

Solution:

$$\begin{aligned}
y' &= 3x^2 \ln x + x^3 \frac{1}{x} \\
&= 3x^2 \ln x + x^2 \\
y'' &= (6x \ln x + 3x^2 \frac{1}{x}) + 2x \\
&= 6x \ln x + 3x + 2x \\
&= 6x \ln x + 5x
\end{aligned}$$

Example

Find $\frac{d^2y}{dx^2}$ for

$$xy + y - x = 4$$

Solution:

$$y + xy' + y' - 1 = 0 \rightarrow y' = \frac{1-y}{x+1}$$
$$y' + y' + xy'' + y'' = 0$$
$$(x+1)y'' = -2y'$$
$$y'' = \frac{-2y'}{x+1}$$
$$y'' = \frac{-2\frac{1-y}{x+1}}{x+1}$$
$$y'' = \frac{-2(1-y)}{(x+1)^2}$$