# Section 13.4
# Second Derivative Test

Dr. Abdulla Eid

College of Science

MATHS 104: Mathematics for Business II

# First Derivative Test

**Theorem**

*(Second Derivative Test) Suppose $f'(c) = 0$, then*

1. *If $f''(c) < 0$ , then $f$ has a local maximum at $c$.*
2. *If $f''(c) > 0$ , then $f$ has a local minimum at $c$.*

Notes:

- If $f''(c) = 0$, then we say the second derivative test is inconclusive! and in that case we need to use the first derivative test.
- This is very useful for Section 13.6.

## Example

Find the local maximum and local minimum (if any) using the second derivative test.

$$f(x) = x^3 - 12x + 1$$

Solution:

We find the derivatives first which are

$$f'(x) = 3x^2 - 12$$
$$f''(x) = 6x$$

To find the critical points, we find where the first derivative equal to zero or does not exist.



$f'(x) = 0$

numerator $= 0$

$3x^2 - 12 = 0$

$x = -2$ or $x = 2$

$f'(x)$ does not exist

denominator $= 0$

$1 = 0$

Always False

No Solution

# Second Derivative Test

$$f''(-2) = 6(-2) = -12 < 0$$

So $x = -2$ is a local maximizer with maximum $f(-2) =$.

$$f''(2) = 6(2) = -2 > 0$$

So $x = 2$ is a local minimizer with minimum $f(2) =$.

## Example

Find the local maximum and local minimum (if any) using the second derivative test.

$$f(x) = 7 - 2x^4$$

Solution:

We find the derivatives first which are

$$f'(x) = -8x^3$$

$$f''(x) = -24x^2$$

To find the critical points, we find where the first derivative equal to zero or does not exist.

$f'(x) = 0$

$\text{numerator} = 0$

$-8x^3 = 0$

$x = 0$

$f'(x)$ does not exist

$\text{denominator} = 0$

$1 = 0$

Always False

No Solution

# Second Derivative Test

$$f''(0) = -246(0) = 0$$

So the second derivative test is inconclusive! and we apply the first derivative test.