Section 14.10
Consumers' and Producers' Surplus
2 Lectures

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MATHS 104: Mathematics for Business II

Motivation Example

Example

Scenario: In the supermarket, you find that candy bar are on sale for 250 Fils. This is good news for you, because you were prepared to pay 300 Fils for them.

The supermarket manager is happy to see you loading them into the basket, because he knows the store's wholesale cost is just 180 Fils a bar.

You are thinking: "Lucky me! I'm paying 250 Fils for a 300 Fils item, so I'm coming out 50 Fils ahead on each bar¡'

he's thinking: "Lucky me! I'm selling these 180-Fils bars for 250 Fils, so I'm coming out 70 Fils ahead on each bar¡'

Both parties feel like they are coming out ahead on the transaction. The term **surplus** is used to describe how much each party gains from the transaction

Before we study the surplus, we need to recall the definition of **equilibrium**.

Consumer Surplus

Recall: The Equilibrium point in the market is the point where the demand function intersect with the supply function. We find that point by solving the equation

$$\text{Demand} = \text{Supply}$$

Question: What does the point of equilibrium in term of economics mean?
Answer: The **equilibrium point** (q_0, p_0) is the point that represent where the stability in the producer–consumer relationship occurs. In short, the price p_0 is the price at which the consumers will purchase the same quantity of a product that producers wish to sell at that price. Sometimes, it best to think about it as the "fair price" or "best price".

Example

Graph the demand and supply curves, find the equilibrium point and analyze the graph.

$$\text{Demand:} \quad p = -50q + 2000$$
$$\text{Supply:} \quad p = 10q + 500$$

Solution: We find first the equilibrium point (q_0, p_0).

$$\begin{aligned} \text{Demand} &= \text{Supply} \\ -50q + 2000 &= 10q + 500 \\ 2000 - 500 &= 10q + 50q \\ 1500 &= 60q \\ q &= 25 \end{aligned}$$

So the equilibrium point is $(q_0, p_0) = (25, 750)$.

Preparing to find the Surplus

Draw a horizontal line at the $p_0 = 750$. We will analyze the suplus with $q = 10$.

Example

1. demand=1500.
2. Supply=600.
3. **Equilibrium Price**=750 (Fair price).

The consumers are thinking: "We would have bought 10 units for 1500, but instead we got them for the equilibrium price of 750! What a deal! We came out 750 ahead on each of these items¡'
The producers are thinking: "We would have sold 10 units for 600, but instead we sold them for the equilibrium price of 750. What a deal! We made an extra 150 on each of these items¡'

Definition

1. **Consumer surplus** is the total amount by which the consumers came out ahead. It's equal to the area between equilibrium and demand.

$$\text{CS} = \int_0^{q_0} (\text{Demand} - p_0)\, dq$$

2. Producer surplus is the total amount by which the producers came out ahead. It's equal to the area between equilibrium and supply.

$$\text{PS} = \int_0^{q_0} (p_0 - \text{Supply})\, dq$$

$$
\begin{aligned}
\text{CS} &= \int_0^{q_0} (\text{Demand} - p_0)\, dq \\
&= \int_0^{25} (-50q + 2000 - 750)\, dx = \int_0^{25} (-50q + 1250)\, dx \\
&= \left[-25q^2 + 1250q\right]_0^{25} \\
&= 15625
\end{aligned}
$$

$$
\begin{aligned}
\text{PS} &= \int_0^{q_0} (p_0 - \text{Supply})\, dq \\
&= \int_0^{25} (750 - (10q + 500))\, dx = \int_0^{25} (250 - 10q)\, dx \\
&= \left[250q - 5q^2\right]_0^{25} \\
&= 3125
\end{aligned}
$$

Example

(Old Final Exam Question) The demand and supply equations for a product are:

$$\text{Demand:} \quad p = 30 - 0.05q^2$$
$$\text{Supply:} \quad p = 21 + 0.04q^2$$

Determine the consumers' and producers' surplus at the market equilibrium.

Solution: We find first the equilibrium point (q_0, p_0).

$$\begin{aligned}
\text{Demand} &= \text{Supply} \\
30 - 0.05q^2 &= 21 + 0.04q^2 \\
9 &= 0.09q^2 \\
0 &= 0.09q^2 - 9 \\
q = 10 \text{ or } q &= -10\,(\text{rejected!})
\end{aligned}$$

So the equilibrium point is $(q_0, p_0) = (10, 25)$.

$$
\begin{aligned}
\text{CS} &= \int_0^{q_0} (\text{Demand} - p_0)\, dq \\
&= \int_0^{10} (30 - 0.05q^2 - 25)\, dx = \int_0^{10} (5 - 0.05q^2)\, dx \\
&= \left[5q - \frac{0.05}{3} q^3\right]_0^{10} \\
&= \frac{100}{3}
\end{aligned}
$$

$$
\begin{aligned}
\text{PS} &= \int_0^{q_0} (p_0 - \text{Supply})\, dq \\
&= \int_0^{10} (25 - (21 + 0.04q^2))\, dx = \int_0^{10} (4 - 0.04q^2)\, dx \\
&= \left[4q - \frac{0.04}{3} q^3\right]_0^{10} \\
&= \frac{80}{3}
\end{aligned}
$$

Example

(Old Final Exam Question) The demand and supply equations for a product are:

$$\text{Demand:} \quad p = 75 - q^2$$
$$\text{Supply:} \quad p = 27 + 2q^2$$

Determine the consumers' and producers' surplus at the market equilibrium.

Solution: We find first the equilibrium point (q_0, p_0).

$$\text{Demand} = \text{Supply}$$
$$75 - q^2 = 27 + 2q^2$$
$$48 = 3q^2$$
$$0 = 3q^2 - 48$$
$$q = 4 \text{ or } q = -4 \, (\text{rejected!})$$

So the equilibrium point is $(q_0, p_0) = (4, 59)$.

$$\begin{aligned} \text{CS} &= \int_0^{q_0} (\text{Demand} - p_0)\, dq \\ &= \int_0^4 (75 - q^2 - 59)\, dx = \int_0^4 (16 - q^2)\, dx \\ &= \left[16q - \frac{1}{3}q^3\right]_0^4 \\ &= \frac{128}{3} \end{aligned}$$

$$\begin{aligned} \text{PS} &= \int_0^{q_0} (p_0 - \text{Supply})\, dq \\ &= \int_0^4 (59 - (27 + 2q^2))\, dx = \int_0^4 (32 - 2q^2)\, dx \\ &= \left[32q - \frac{2}{3}q^3\right]_0^4 \\ &= \frac{256}{3} \end{aligned}$$

Exercise

(Old Final Exam Question) The demand and supply equations for a product are:

$$\text{Demand:} \quad p = 20 - 0.06q^2$$
$$\text{Supply:} \quad p = 11 + 0.03q^2$$

Determine the consumers' and producers' surplus at the market equilibrium.

Solution: We find first the equilibrium point (q_0, p_0).

$$\text{Demand} = \text{Supply}$$
$$20 - 0.06q^2 = 11 + 0.03q^2$$
$$9 = 0.09q^2$$
$$0 = 0.09q^2 - 9$$
$$q = 10 \text{ or } q = -10\,(\text{rejected!})$$

So the equilibrium point is $(q_0, p_0) = (10, 14)$.

$$
\begin{aligned}
\text{CS} &= \int_0^{q_0} (\text{Demand} - p_0)\, dq \\
&= \int_0^{10} (20 - 0.06q^2 - 14)\, dx = \int_0^{10} (6 - 0.06q^2)\, dx \\
&= \left[6q - \frac{0.06}{3} q^3 \right]_0^{10} \\
&= 40
\end{aligned}
$$

$$
\begin{aligned}
\text{PS} &= \int_0^{q_0} (p_0 - \text{Supply})\, dq \\
&= \int_0^{10} (14 - (11 + 0.03q^2))\, dx = \int_0^{10} (3 - 0.03q^2)\, dx \\
&= \left[3q - \frac{0.03}{3} q^3 \right]_0^{10} \\
&= 20
\end{aligned}
$$

Example

(Old Final Exam Question) The demand and supply equations for a product are:

$$\text{Demand:} \quad p = 25 - 0.2q^2$$
$$\text{Supply:} \quad p = 5 + 3q$$

Determine the consumers' and producers' surplus at the market equilibrium.

Solution: We find first the equilibrium point (q_0, p_0).

$$\begin{aligned} \text{Demand} &= \text{Supply} \\ 25 - 0.2q^2 &= 5 + 3q \\ 0 &= 0.2q^2 + 3q - 20 \\ q = 5 \text{ or } q &= -20 \, (\text{rejected!}) \end{aligned}$$

So the equilibrium point is $(q_0, p_0) = (5, 20)$.

$$
\begin{aligned}
\text{CS} &= \int_0^{q_0} (\text{Demand} - p_0)\, dq \\
&= \int_0^5 (25 - 0.2q^2 - 20)\, dx = \int_0^5 (5 - 0.2q^2)\, dx \\
&= \left[5q - \frac{0.2}{3} q^3\right]_0^5 \\
&= \frac{50}{3}
\end{aligned}
$$

$$
\begin{aligned}
\text{PS} &= \int_0^{q_0} (p_0 - \text{Supply})\, dq \\
&= \int_0^5 (20 - (5 + 3q))\, dx = \int_0^5 (15 - 3q)\, dx \\
&= \left[15q - \frac{3}{2} q^2\right]_0^5 \\
&= \frac{75}{2}
\end{aligned}
$$

Example

(Old Final Exam Question) The demand of a product is

$$\text{Demand:} \quad q = 10\sqrt{100 - p}$$

Determine the consumers' surplus at the market equilibrium which occurs when $p = 84$.

Solution: We find first the equilibrium point (q_0, p_0). Now $q_0 = 10\sqrt{100 - 84} = 40$. Moreover, we re–write the function to get

$$q^2 = 100(100 - p)$$

$$\frac{q^2}{100} = 100 - p$$

$$p = 100 - \frac{1}{100}q^2$$

$$
\begin{aligned}
\text{CS} &= \int_0^{q_0} (\text{Demand} - p_0)\, dq \\
&= \int_0^{40} \left(100 - \frac{1}{100} q^2 - 84\right) dx = \int_0^{40} \left(16 - \frac{1}{100} q^2\, dq \right.\\
&= \left[16q - \frac{1}{300} q^3\right]_0^{40} \\
&= \frac{1280}{3}
\end{aligned}
$$

Significance of Consumer Surplus

1- When Consumer Surplus is high:
This means that consumers have more money 'left over' to spend than they were expecting. They were prepared to pay higher prices for items they needed, and because they didn't have to, they have money left over (more saving in hand). Some of the effects of a high consumer surplus are:

- Lower crime rate
- Increased sales of non-necessities: vacation travel goes up, movie attendance goes up, etc.

Significance of Producer Surplus

2- When Producer Surplus is high:
This means that producers have more profits than they were expecting. They were able to sell their products for a higher price than they were willing to sell them for. Some of the effects of a high producer surplus are:

- More money invested in product research (resulting in better, safer, more efficient products.)
- More money for company growth and expansion (resulting in more jobs and lower unemployment.)

Reference

- Lecture notes of Margaret McQuain which can be found online (retrieved April 17, 2016) at

  http://www.math.vt.edu/people/mcquain/csps.pdf