

# Section 14.2

## Indefinite Integrals

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# Indefinite Integral

Given a function  $f$ , if  $F$  is a function such that

$$F'(x) = f(x)$$

then  $F$  is called **antiderivative** of  $f$ .

## Definition

An antiderivative of  $f$  is simply a function whose derivative is  $f$ .

## Example

Let  $f(x) = 2x$ , then  $F(x) = x^2$ ,  $F(x) = x^2 + 1$ ,  $F(x) = x^2 + 6$ , and  $F(x) = x^2 + C$ , etc are all **antiderivatives** of  $f(x)$

**Note:** Any two antiderivatives of a function differ only by a constant.

# Indefinite Integrals

If  $F(x)$  is the antiderivative of  $f(x)$ , we will write

$$\int f(x) dx = \underbrace{F(x)}_{\text{antiderivative}} + C$$

where

- The symbol  $\int$  is called the **integral sign**.
- The function  $f(x)$  is called the **integrand**.
- The constant  $C$  is called the **constant of integration**.
- $dx$  indicates the variable involved in the integration which is  $x$ .

**Note:**

$$\frac{d}{dx} \left( \int f(x) dx \right) = f(x) \text{ and } \int \frac{d}{dx} (f(x)) dx = f(x)$$

Integration and differentiation are reversing each other.

## Examples

### Example

Find  $\int 7 dx$ .

Solution: We need to find what is the function that if we differentiate it we get 7?

$$\int 7 dx = 7x + C$$

### Example

Find  $\int x dx$ .

Solution: We need to find what is the function that if we differentiate it we get  $x$ ?

$$\int x dx = \frac{1}{2}x^2 + C$$

## Examples

### Example

Find  $\int x^9 dx$ .

Solution: We need to find what is the function that if we differentiate it we get  $x^9$ ?

$$\int x^9 dx = \frac{1}{10}x^{10} + C$$

### Example

Find  $\int \frac{1}{x} dx$ .

Solution: We need to find what is the function that if we differentiate it we get  $\frac{1}{x}$ ?

$$\int \frac{1}{x} dx = \ln x + C$$

## Elementary integration formula

$$\textcircled{1} \int k \, dx = kx + C.$$

$$\textcircled{2} \int x^n \, dx = \frac{1}{n+1} x^{n+1} + C \quad n \neq -1.$$

$$\textcircled{3} \int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln x + C \quad x > 0.$$

$$\textcircled{4} \int e^x \, dx = e^x + C .$$

$$\textcircled{5} \int kf(x) \, dx = k \int f(x) \, dx .$$

$$\textcircled{6} \int (f(x) + g(x)) \, dx = \int f(x) \, dx + \int g(x) \, dx .$$

### Example

Find  $\int 5x^{-9} dx$ .

Solution:

$$\int 5x^{-9} dx = \frac{5}{-8}x^{-8} + C$$

### Example

Find  $\int \frac{3}{x^6} dx$ .

Solution:

$$\int \frac{3}{x^6} dx = \int 3x^{-6} dx = \frac{3}{-5}x^{-5} + C$$

### Example

Find  $\int 4x^6 + 3x^4 + 2x + 9 + \frac{1}{x} dx$ .

Solution:

$$\int 4x^6 + 3x^4 + 2x + 9 + \frac{1}{x} dx = \frac{4}{7}x^7 + \frac{3}{5}x^5 + x^2 + 9x + \ln x + C$$

### Example

Find  $\int x^{9.9} - 7x^6 + 3x^{-4} + x^{-1} + \sqrt{2} dx$ .

Solution:

$$x^{9.9} - 7x^6 + 3x^{-4} + x^{-1} + \sqrt{2} = \frac{1}{10.9}x^{10.9} - x^7 + \frac{3}{-3}x^{-3} + \ln x + \sqrt{2}x + C$$



### Example

Find  $\int \sqrt{x} + \frac{5}{3\sqrt[3]{x^2}} dx$ .

Solution:

$$\int \sqrt{x} + \frac{5}{3\sqrt[3]{x^2}} dx = \int x^{\frac{1}{2}} + \frac{5}{3}x^{-\frac{2}{3}} dx = \frac{2}{3}x^{\frac{3}{2}} + \frac{5}{3}3x^{\frac{1}{3}} + C$$

### Example

Find  $\int e^x + x^e + e^2 dx$ .

Solution:

$$\int e^x + x^e + e^2 dx = e^x + \frac{1}{e+1}x^{e+1} + e^2x + C$$

### Example

Find  $\int x^{-2}(4x^3 + 3x + 5) dx$ .

Solution:

$$\int x^{-2}(4x^3 + 3x + 5) dx = \int 4x + 3x^{-1} + 5x^{-2} dx = 2x + 3 \ln x - 5x^{-1} + C$$

### Example

Find  $\int \frac{x^4 + 10x}{x^2} dx$ .

Solution:

$$\int \frac{x^4 + 10x}{x^2} dx = \int x^2 + 10x^{-1} dx = \frac{1}{3}x^3 + 10 \ln x + C$$

### Example

Find  $\int (x + 2)^2 dx$ .

Solution:

$$\int (x + 2)^2 dx = \int x^2 + 4x + 4 dx = \frac{1}{3}x^3 + 2x^2 + 4x + C$$

### Example

Find  $\int \frac{d}{dx} \left( \frac{1}{\sqrt{1+x^3}} \right) dx$ .

Solution:

$$\int \frac{d}{dx} \left( \frac{1}{\sqrt{1+x^3}} \right) dx = \frac{1}{\sqrt{1+x^3}} + C$$

### Example

(Old Exam Question) Find  $\int (7x^3 - 6x^2 - \ln 3) dx$ .

Solution:

$$\int (7x^3 - 6x^2 - \ln 3) dx = \frac{7}{4}x^4 - 2x^3 - (\ln 3)x + C$$

### Example

(Old Exam Question) Find  $\int \frac{-3}{x^{3.3}} dx$ .

Solution:

$$\int \frac{-3}{x^{3.3}} dx = \int -3x^{-3.3} dx = \frac{-3}{-2.3}x^{-2.3} + C$$

### Example

(Old Exam Question) Find  $\int (8 - 5e^x) dx$ .

Solution:

$$\int (8 - 5e^x) dx = 8x - 5e^x + C$$

### Example

(Old Exam Question) Find  $\int (\sqrt{x} - \frac{3}{x}) dx$ .

Solution:

$$\int (\sqrt{x} - \frac{3}{x}) dx = \int (x^{\frac{1}{2}} - \frac{3}{x}) dx = \frac{2}{3}x^{\frac{3}{2}} - 3 \ln x + C$$

## Exercise

(Old Exam Question) Find  $\int \left(9x^{3.5} - \frac{5}{x}\right) dx$ .

Solution:

$$\int \left(9x^{3.5} - \frac{5}{x}\right) dx = \frac{9}{4.5}x^{4.5} - 5 \ln x + C$$

## Exercise

(Old Final Exam Question) Find  $\int x^{-0.9} dx$ .

Solution:

$$\int x^{-0.9} dx = \frac{1}{0.1}x^{0.1} + C$$

## Exercise

(Old Exam Question) Find  $\int (1 - 2x + x^3) dx$ .

Solution:

$$\int (1 - 2x + x^3) dx = x - x^2 + \frac{1}{4}x^4 + C$$

## Exercise

(Old Final Exam Question) Find  $\int x^{-1.8} dx$ .

Solution:

$$\int x^{-1.8} dx = \frac{1}{-0.8} x^{-0.8} + C$$