

Section 17.1
Partial Derivative
0.25 Lecture

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MATHS 104: Mathematics for Business II

The Multivariable function

Definition

A Multivariable function is a function that takes more than **one** input. Usually, we write it as $f(x, y)$ or $f(x, y, z)$, etc.

Example

The demand on the cars depends on

- 1 The price of the car.
- 2 The price of the gas.
- 3 The price of maintenance and auto parts.

Example

- 1 $f(x, y) = x^2 + y^2$.
- 2 $f(x, y) = e^{xy} - y^2 + xy + 7$.
- 3 $f(x, y) = \ln y + x^2$.
- 4 $f(x, y, z) = x^7 y z^2 + 3z^4 y$.

The Partial Derivative

Definition

If $z = f(x, y)$, the **partial derivative with respect to x** is given by

$$f_x(x, y) = \lim_{h \rightarrow 0} \frac{f(x + h, y) - f(x, y)}{h}$$

Similarly, the **partial derivative with respect to y** is given by

$$f_y(x, y) = \lim_{h \rightarrow 0} \frac{f(x, y + h) - f(x, y)}{h}$$

Procedure to find f_x and f_y :

- To find f_x , treat y as a constant and differentiate in the usual way with respect to x .
- To find f_y , treat x as a constant and differentiate in the usual way with respect to y .

Example

If $f(x, y) = 2x^2 + 3xy$. Find f_x and f_y .

Solution:

$$f_x = 4x + 3y$$

$$f_y = 3x$$

Example

If $f(x, y) = e^{xy}$. Find f_x and f_y .

Solution:

$$f_x = e^{xy} y$$

$$f_y = e^{xy} x$$

Notation: Let $z = f(x, y)$

- f_x is denoted by $\frac{\partial f}{\partial x}$.
- f_y is denoted by $\frac{\partial f}{\partial y}$.

Example

Let $f(x, y) = \frac{x}{y}$. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

Solution:

$$\frac{\partial f}{\partial x} = \frac{1}{y}$$

$$\frac{\partial f}{\partial y} = \frac{-x}{y^2}$$

Example

If $f(x, y, z) = 2x^3y^2 + 2xy^3z + 4z^2$. Find f_x , f_y and f_z .

Solution:

$$f_x = 6x^2y^2 + 2y^3z$$

$$f_y = 4x^3y + 6xy^2z$$

$$f_z = 2xy^3 + 8z$$