Section 3.7
Implicit Differentiation
1 Lecture

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MATHS 101: Calculus I

Implicitly Defined Functions

So far, we have seen explicitly defined functions, i.e., functions of the form $y = f(x)$, where y is expressed totally in terms of x. Now we want to deal with implicitly defined functions such as the following:

Example

1. $x^2 + y^2 = 4$.
2. $e^{xy} - x^2 + 4y = 5x - 4$.
3. $\ln(y) + e^{2x} = y^2 e^{-x}$.

The above functions are said to be implicitly defined function. Note that sometimes, it is hard (or even impossible) to write y alone as a function of x.

Goal: To find the derivative y' of implicitly defined functions.

Example

Find y' for

$$y^2 = x$$

Solution 1:

Solution 2:

Exercise

Find $\frac{dy}{dx}$ for $xy = 1$ by two ways, implicit and explicit.

Example

Find y' for

$$x^2 + y^2 = 4$$

and use it to find an equation of the tangent line at $(1, \sqrt{3})$.

Solution: To find the derivative y', we differentiate both sides with respect to x to get

Hence the equation of the tangent line is

$$y - y_1 = m(x - x_1) \rightarrow$$

Example

Find y' for

$$xy^2 - 6x = 5 + 2y$$

Solution:

Example

Find y' for

$$x + e^y = y + e^x$$

Solution:

Example

Find y' for

$$xy = \cot(y)$$

Solution:

Exercise

Find $\frac{dy}{dx}$ for $\sin(xy) = \frac{1}{2}$.

Example

(The Folium of Descarte')

(a) Find y' if $x^3 + y^3 = 6xy$.

(b) Find an equation of the tangent line at $(3, 3)$.

(c) Find an equation of the normal line at $(3, 3)$.

Solution: (a)

Continue...

Example

(The Folium of Descarte')

(a) Find y' if $x^3 + y^3 = 6xy$.

(b) Find an equation of the tangent line at $(3, 3)$.

Solution: (b) The equation of the tangent line is given by

We find the slope at $(,)$

Continue...

Hence we have

$$y - y_1 = m(x - x_1)$$

Example

Find an equation of the tangent and normal lines at $(0, \pi)$ for the curve

$$x^2 \cos^2 y - \sin y = 0$$

Solution: Hence the equation of the tangent line is

$$y - y_1 = m(x - x_1) \rightarrow$$

Hence the equation of the normal line is

Example

Find $\frac{d^2y}{dx^2}$ for

$$xy + y - x = 4$$

Solution:

$$\rightarrow$$