# Section 3.8 Derivative of the inverse function and logarithms 3 Lecture

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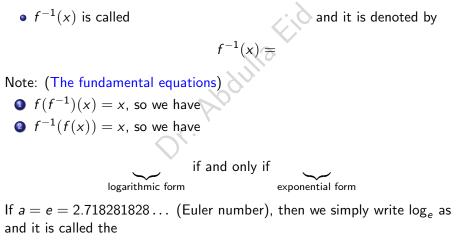
MATHS 101: Calculus I

## Topics

- **1** Inverse Functions (1 lecture).
- 2 Logarithms.
- Orivative of the inverse function (1 lecture).
- Logarithmic differentiation (1 lecture).

# 2- Logarithmic Function

Consider the exponential function  $f(x) = a^x$ . Question: Does f(x) has an inverse? Why? Answer:



## Properties of Logarithms

- $0 \ \log_a(m \cdot n) = .$
- $\log_a(\frac{m}{n}) = .$

- (change of bases)  $\log_a m = 1$

#### Exercise

Use the fundamental equations to prove these six properties of the logarithms.

(Expansion) Write the following expression as sum or difference of logarithms

In
$$(\frac{x}{wz^2}) =$$
In $(\frac{x+1}{x+5})^4 =$ 
In $(\frac{\sqrt{x}}{(x^2)(x+3)^4})$ 

#### Exercise

Write each of the following expression as sum or difference of logarithms: (1)  $\log_3(\frac{5\cdot7}{4})$  (2)  $\log_2(\frac{x^5}{y^2})$  (3)  $\log(\frac{x^2z}{wy^2})$  (4)  $\ln\sqrt{\frac{x+1}{x-2}}$ .

Write each of the following logarithm in terms of natural logarithm.

- **1**  $\log_3 x =$
- log<sub>6</sub> 7 =
- $\log_2 y =$

The derivative of the inverse function

Strategy: Goal: We want to find  $\frac{d}{dx}(f^{-1}(x))$ . Write  $y = f^{-1}(x)$ , we want to find y'

### Geometric Interpretation \*

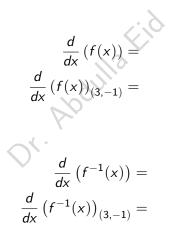
Note that

$$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'(f^{-1}(x))}$$

so the slope of  $f^{-1}$  is reciprocal to the slope of f. Geometrically,

Let  $f(x) = x^3 - 3x^2 - 1$ . Find  $\frac{d}{dx}(f(x))$  and  $\frac{d}{dx}(f^{-1}(x))$  at the point (3, -1)

Solution:



Derivative of In

Example

Find  $\frac{d}{dx}(\ln x)$ .

Solution:

 $y = \ln x$ 

#### Exercise

Find y' if  $y = \log_a x$ . (Hint: Use the change of base formula to change it to ln)

### Recall

The Chain Rule

Theorem

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

 $(f(g(x)))' = derivative of outer(inner) \cdot (derivative of inner)$ 

Find y' for each of the following:

**a** 
$$f(x) = \ln x^2 = \ln x^2 \rightarrow y' =$$
**b**  $f(x) = \ln(2x+3) = \ln(2x+3) \rightarrow y' =$ 
**a**  $f(x) = x \ln x \rightarrow y' =$ 
**b**  $f(x) = \ln(\ln x) = \ln(\ln x) \rightarrow y' =$ 
**b**  $f(x) = \ln(\sin x) = \ln(\sin x) \rightarrow y' =$ 
**c**  $f(x) = \sin(\ln x) = \sin(\ln x) \rightarrow y' =$ 
**c**  $f(x) = \sin(\ln x) = \sin(\ln x) \rightarrow y' =$ 

Derivative using the properties of Logarithms

#### Example

Find the derivative of

•  $f(x) = \ln x^{2017}$ 

Solution: First we re-write the function in terms using the properties of the ln to get a simplified function:

$$f(x) =$$

Hence

f'(x) =

#### Exercise

Using the chain rule, find the derivative of the function of the previous example without using the properties of the ln, i.e., find f'(x) for

 $f(x) = \ln(x^{2017})$ 

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## Derivative using the properties of Logarithms

#### Example

Find the derivative of

• 
$$f(x) = \ln \sqrt[3]{\frac{x^3 - 1}{x^3 + 1}}$$

Solution: First we re-write the function in terms using the properties of the ln to get a simplified function:

$$f(x) = \ln\left(\frac{x^3 - 1}{x^3 + 1}\right)^{\frac{1}{3}}$$

### Continue...

We write the inner function in blue and the outer function in red and we apply the chain rule.

derivative of outer (inner)  $\cdot$  (derivative of inner)

f(x) = f'(x) =

#### Exercise

Using the chain rule, find the derivative of the function of the previous example without using the properties of the ln, i.e., find f'(x) for

$$f(x) = \ln\left(\sqrt[3]{\frac{x^3 - 1}{x^3 + 1}}\right)$$



