Section 3.8
Derivative of the inverse function and logarithms
3 Lectures

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MATHS 101: Calculus I

Topics

1. Inverse Functions (1 lecture).
2. Logarithms.
3. Derivative of the inverse function (1 lecture).
4. Logarithmic differentiation (1 lecture).

Logarithmic Differentiation

Goal: To find the derivative of $y = f(x)$, where $f(x)$ is possibly involving quotient, product, powers, etc.

Example

1. $y = \dfrac{(x+1)^4(3x^2+5)}{(4x-5)\sqrt[4]{4x^2+5}}$.
2. $y = \left(\dfrac{(x+5)(4x-2)^7}{x^2+5x+2}\right)^5$.
3. $y = x^{\sqrt{x}}$. — variable$^{\text{variable}}$.
4. $y = \ln x^{x^2+3x+5}$.

Idea

To differentiate $y = f(x)$,

1. Take the natural logarithm of both sides to get

$$\ln y = \ln\left(f(x)\right)$$

2. Simplify $\ln(f(x))$ by using the properties of the logarithms.
3. Differentiate both sided with respect to x.
4. Solve for y'.
5. Express the answer in term of x (substitute $f(x)$ for y).

Example

Find y' for

$$y = x^{x+1}$$

Solution:
We take In of both sides to get and We simplify the right hand side using the properties of logarithms to get

Example

Find y' for

$$y = (x)^{\sin x}$$

Solution:

Example

Find y' for

$$y = (3x)^{\sqrt{x}}$$

Solution:

We take ln of both sides to get and We simplify the right hand side using the properties of logarithms to get

Exercise

Find y' for

$$y = (\ln x)^{\ln x}$$

Example

(General Form) Find y' for

$$y = f(x)^{g(x)}$$

Solution:
We take ln of both sides to get and We simplify the right hand side using the properties of logarithms to get

Example

Find y' for

$$y = \sqrt{\frac{5-4x}{1+x^2}}$$

Solution:

Example

Find y' for

$$y = \frac{(1-2x)^3(4+5x^6)^7}{\sqrt[3]{8-9x}}$$

Solution: