

University of Bahrain
Department of Mathematics
MATHS101: Calculus I
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Worksheet: Continuity

Students' Name: _____

1. Is the following function

$$f(x) = \begin{cases} \frac{3x+1}{x+2}, & x \neq 2 \\ 5, & x = 2 \end{cases}$$

continuous at $x = 2$?

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2. Consider

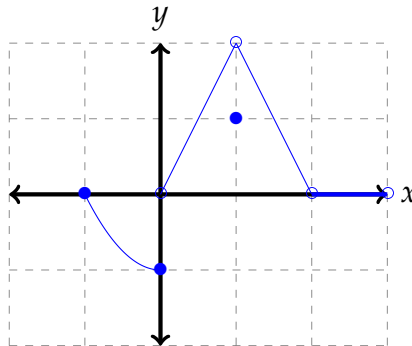
$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & 1.5 < x < 2 \\ 5x^2 + 1, & x \geq 2 \\ \frac{x^2 - 1}{x - 1}, & x \neq 1 \\ 3, & x < 0 \end{cases}$$

1. Is the function continuous at $x = 2$

2. Is the function continuous at $x = 1$

3. Is the function continuous at $x = 0$

3. Consider the following function:



1. Does $f(-1)$ exist?
2. Is the function continuous at $x = -1$
3. Is the function continuous at $x = 1$
4. Is the function continuous at $x = 2$
5. Is the function continuous at $x = 0$
6. In which interval the function is continuous?
7. What should be the value of $f(2)$ for the function to be continuous?

4. For what value(s) of a is the function

$$f(x) = \begin{cases} a^2x - 2a, & x \geq 2 \\ 12, & x < 2 \end{cases}$$

continuous at $x = 2$.

5. Find the interval where the following functions are continuous:

1. $f(x) = \frac{x^2 - 1}{3x^2 - 5x - 2}$

2. $f(x) = \frac{3}{\sqrt{x-4}}$

6. Show there exists a root for $x^3 - 3x - 1 = 0$.

7. If f is continuous function at a and g is continuous function at $b = f(a)$, then the composite $g \circ f$ is continuous function at a .

(Hint: Compute $\lim_{x \rightarrow a} (g \circ f)(x)$)

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8. (Challenging problem) Show whether the following function is continuous or not at any number of your choice.

$$f(x) = \begin{cases} 0, & x \text{ is rational} \\ 1, & x \text{ is irrational} \end{cases}$$

9. (Challenging Problem) **The fixed point theorem** Suppose f is a continuous function on $[0, 1]$ such that $0 \leq f(x) \leq 1$. Show there exist $c \in (0, 1)$ such that $f(c) = c$.
(Hint: Apply IVT to $g(x) = f(x) - x$).

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