



3. (At home) In this exercise, you should prove that the sine function is continuous.

1. Prove that  $f$  is continuous at  $a$  if and only if

$$\lim_{h \rightarrow 0} f(a + h) = f(a)$$

2. Use the trigonometric identities to show that

$$\lim_{h \rightarrow 0} \sin(a + h) = \sin(a)$$

and use the exercise above to show that  $f(x) = \sin x$  is a continuous function. Do the same for the  $f(x) = \cos x$ .

4. Find  $\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta}$

5. Find  $\lim_{x \rightarrow 0^+} \sqrt{x} e^{\sin\left(\frac{\pi}{x}\right)}$

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6. For which value(s) of  $k$  is the function defined by

$$f(x) = \begin{cases} \frac{\sin(2x)}{x}, & x < 0 \\ \cos x + x^2 + 4k, & x \geq 0 \end{cases}$$

continuous at  $x = 0$ ?