Section 3.3
Constant Multiple and sum rule
1 Lecture

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MATHS 101: Calculus I

Constant Factor Rule

Theorem 1

$$(cf(x))' = cf'(x)$$

Let $F(x) = cf(x)$.

$$\begin{aligned}
\frac{d}{dx}(c \cdot f(x)) = F'(x) &= \lim_{h \to 0} \frac{F(x+h) - F(x)}{h} \\
&= \lim_{h \to 0} \frac{c \cdot f(x+h) - (c \cdot f(x))}{h} \\
&= \lim_{h \to 0} \frac{c \cdot (f(x+h) - f(x))}{h} \\
&= c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= cf'(x)
\end{aligned}$$

Example 2

1. $\frac{d}{dx}\left(5x^2\right) = 5\frac{d}{dx}\left(x^2\right) = 5 \cdot 2x = 10x.$
2. $\frac{d}{dx}\left(\frac{8}{x^5}\right) = 8\frac{d}{dx}\left(\frac{1}{x^5}\right) = 8\frac{d}{dx}\left(x^{-5}\right) = -40x^{-6}.$
3. $\frac{d}{dx}\left(7x^3\sqrt[4]{x}\right) = 7\frac{d}{dx}\left(x^3x^{\frac{1}{4}}\right) = 7\frac{d}{dx}\left(x^{\frac{13}{4}}\right) = -\frac{7\cdot 13}{4}x^{-\frac{9}{4}}.$

Sum Rule

Theorem 3

$$\frac{d}{dx}\left(f(x)+g(x)\right)=\frac{d}{dx}\left(f(x)\right)+\frac{d}{dx}\left(g(x)\right)$$

Let $F(x)=f(x)+g(x)$.

$$\begin{aligned}
\frac{d}{dx}\left(f(x)+g(x)\right)=F'(x) &= \lim_{h\to 0}\frac{F(x+h)-F(x)}{h}\\
&= \lim_{h\to 0}\frac{f(x+h)+g(x+h)-(f(x)+g(x))}{h}\\
&= \lim_{h\to 0}\frac{f(x+h)+g(x+h)-f(x)-g(x)}{h}\\
&= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\\
&= \frac{d}{dx}\left(f(x)\right)+\frac{d}{dx}\left(g(x)\right)
\end{aligned}$$

Example 4

Find y' and simplify:

1. $y = \frac{x^3}{3} - \frac{2}{x^2}$.
2. $y = x^2(4x+6)$.
3. $y = \frac{x^8+x^5}{x^2}$.
4. $y = \sqrt{2} + e^{\sqrt{2}} + \ln\sqrt{2}$.
5. $x^3 - \ln 2$.

Solution:

1. $y = \frac{1}{3}x^3 - 2x^{-2} \rightarrow y' = \frac{3}{3}x^2 + 4x^{-3} = x^2 + \frac{4}{x^3}$.
2. $y = 4x^3 + 6x^2 \rightarrow y' = 12x^2 + 12x$.
3. $y = x^6 + x^3 \rightarrow y' = 6x^5 + 3x^2$.
4. $y' = 0$. Since all functions are constant functions.
5. $y' = 3x^2$.

Example 5

Find all the points on the curve $y = x^3 - 3x + 6$ where the slope of the tangent line is 9.

Solution: Recall that the slope of the tangent line is the derivative, we need to find the derivative and make it equal to 9.

$$\begin{aligned} \text{Slope of the tangent line} &= 9 \\ f'(x) &= 9 \\ 3x^2 - 3 &= 9 \\ 3x^2 - 12 &= 0 \\ x &= 2 \text{ or } x = -2 \end{aligned}$$

The points are then

$$(2, 8) \text{ and } (-2, 4)$$

Example 6

Find an equation of the tangent line to the curve $f(x) = 5 - \sqrt[4]{x}$ at $x = 1$.

Solution: Recall that an equation of a line at (x_1, y_1) and slope m is

$$y - y_1 = m(x - x_1)$$

$$m = \text{Slope of the tangent line} = m = f'(1)$$

$$f(x) = 5 - x^{\frac{1}{4}} \to f'(x) = -\frac{1}{4}x^{-\frac{5}{4}}$$

$$m = f'(1) = \frac{-1}{4}$$

Now $x_1 = 1$ and $y_1 = f(1) = 4$. Hence the equation of the tangent line is

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{-1}{4}(x - 1)$$

$$-4y + 16 = x - 1$$

$$-4y - x = -17$$

Exercise 7

Find an equation of the tangent line to the curve $f(x) = \frac{\sqrt{x}(2-x^2)}{x}$ at $x = 4$.