Section 4.1
Absolute Extrema
1/2 Lecture

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MATHS 101: Calculus I

Extreme Value Theorem

Recall: To find the local max or local min, we need to apply the first derivative test.

Theorem 1

(Extreme Value Theorem) If f is a continuous function of a closed interval $[a, b]$, then it has both a global maximum and global minimum.

The theorem above guarantees that we have a global max and global min. The question is how to find the global max and global min?

1. Find the critical points c and evaluate $f(c)$.
2. Find the value of the function at the endpoints $f(a), f(b)$.
3. The global maximum (or global minimum) is the one that is the largest (smallest) value.

Example 2

Find the absolute maximum and absolute minimum values of

$$f(x) = x^3 - 3x + 5$$

on the interval $[-3, 0]$

Solution:
We find the derivative first which is

$$f'(x) = 3x^2 - 3$$

To find the critical points, we find where the derivative equal to zero or does not exist.

$$\begin{aligned} f'(x) &= 0 \\ \text{numerator} &= 0 \\ 3x^2 - 3 &= 0 \\ x = 1 &\text{ or } x = -1 \end{aligned}$$

$$f'(x) \text{ does not exist}$$

$$\begin{aligned} \text{denominator} &= 0 \\ 1 &= 0 \end{aligned}$$

Always False

No Solution

Table

Now we fill the table with critical points as well as the endpoints

x	-3	-1	0
$f(x)$			

1. The absolute minimum of f is -13 at $x = -3$.
2. The absolute maximum of f is 7 at $x = -1$.

Exercise 3

Find the absolute maximum and absolute minimum values of

$$f(x) = 3x^4 - 4x^3$$

on the interval [-2,2]

Example 4

Find the absolute maximum and absolute minimum values of

$$f(x) = x + \frac{1}{x}$$

on the interval $[0.5, 4]$

Solution:
We find the derivative first which is

$$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$$

To find the critical points, we find where the derivative equal to zero or does not exist.

$$\begin{aligned} f'(x) &= 0 \\ \text{numerator} &= 0 \\ x^2 - 1 &= 0 \\ x &= 1 \text{ or } x = -1 \end{aligned} \qquad\qquad \begin{aligned} f'(x) & \text{ does not exist} \\ \text{denominator} &= 0 \\ x^2 &= 0 \\ x &= 0 \end{aligned}$$

Table

Now we fill the table with critical points as well as the endpoints

x	0.5	1	4
$f(x)$			

1. The absolute maximum of f is 4.25 at 4.
2. The absolute minimum of f is 2 at 1.

Exercise 5

Find the absolute maximum and absolute minimum values of

$$f(x) = x^{-2} \ln x$$

on the interval $[0.5, 4]$

Example 6

Find the absolute maximum and absolute minimum values of

$$f(x) = t\sqrt{4-t^2}$$

on the interval $[-1, 2]$

Solution:

We find the derivative first which is

$$f'(x) = \sqrt{4-t^2} + t \cdot \frac{1}{2\sqrt{4-t^2}} \cdot -2t = \sqrt{4-t^2} - \frac{t^2}{\sqrt{4-t^2}} = \frac{4-2t^2}{\sqrt{4-t^2}}$$

To find the critical points, we find where the derivative equal to zero or does not exist.

$$f'(x) = 0$$

$$\text{numerator} = 0$$

$$4 - 2t^2 = 0$$

$$x = -\sqrt{2} \text{ or } x = \sqrt{2}$$

$$f'(x) \text{ does not exist}$$

$$\text{denominator} = 0$$

$$4 - t^2 = 0$$

$$x = -2 \text{ or } x = 2$$

Table

Now we fill the table with critical points as well as the endpoints

x	-1	$\sqrt{2}$	2
$f(x)$			

1. The absolute maximum of f is 2 at $\sqrt{2}$.
2. The absolute minimum of f is $-\sqrt{3}$ at -1.

Exercise 7

Find the absolute maximum and absolute minimum values of

$$f(x) = \cos x$$

on the interval $\left[-\frac{\pi}{2}, \frac{-\pi}{4}\right]$