Section 4.2
The mean value theorem
1 Lecture

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MATHS 101: Calculus I

Rolle's Theorem

Theorem 1

Let $f(x)$ be a function on the interval $[a, b]$ such that the following hypothesis are satisfied:

1. $f(x)$ *is continuous on* $[a, b]$.
2. $f(x)$ *is differentiable on* (x, b).
3. $f(a) = f(b)$.

Then there exists $c \in (a, b)$ *such that*

$$f'(c) = 0$$

Example 2

Determine all the numbers c which satisfy the conclusions of the Rolle's Theorem for the following function

$$f(x) = -x^3 - x^2 + 2x \qquad \text{on } [-2, 1]$$

Mean Value Theorem

Theorem 3

Let $f(x)$ be a function on the interval $[a, b]$ such that the following hypothesis are satisfied:

1. *$f(x)$ is continuous on $[a, b]$.*
2. *$f(x)$ is differentiable on (x, b).*

Then there exists $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Example 4

Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function

$$f(x) = x^3 + 2x - x \qquad \text{on } [-1, 2]$$

Example 5

Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function

$$f(x) = 8t + e^{-3t} \qquad \text{on } [-2, 3]$$

Example 6

Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function

$$f(x) = 9x - 8\sin\left(\frac{x}{2}\right) \qquad \text{on } [-3, 1]$$

Example 7

Suppose that $f(x)$ is continuous and differentiable on $[6, 15]$. Suppose that $f(6) = -1$ and $f'(x) \leq 10$ for all $x \in (6, 15)$. Find the largest possible value of $f(15)$?