Kingdom of Bahrain
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Pythagoras and His School
P.37&38

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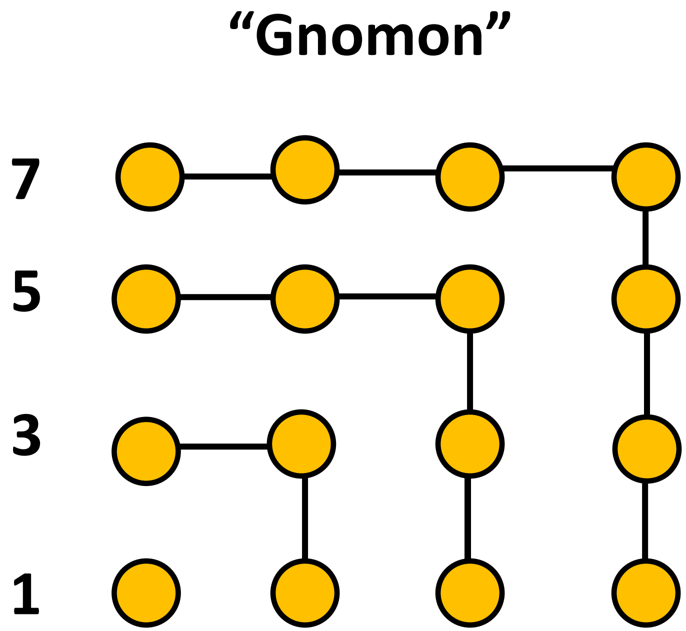
Year 3 - Section 4
2014 - 2015

Dot representation for square numbers:



How many dots are added in each row?

Consecutive odd numbers are added each time.

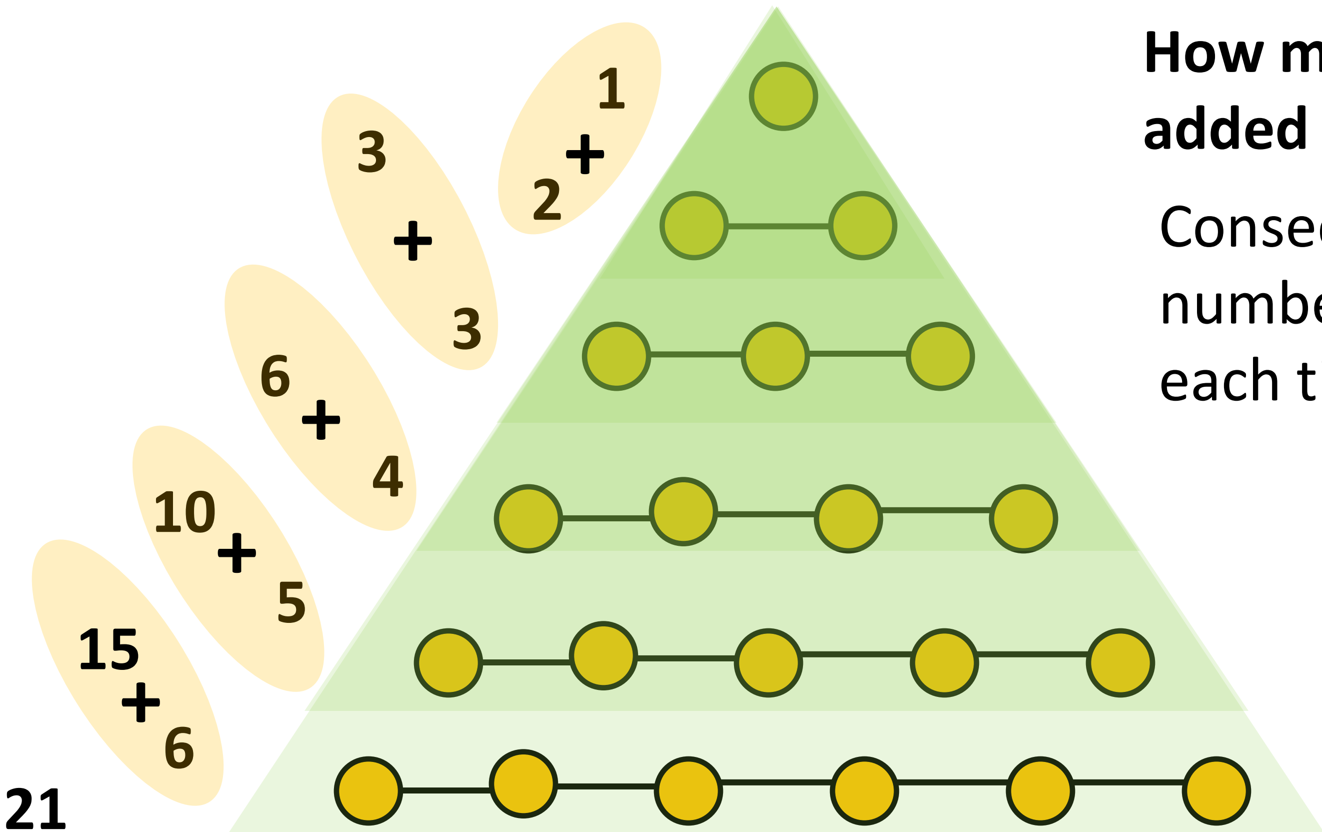
1×1=1 **2×2=4** **3×3=9** **4×4=16**

1 **1+3=4** **1+3+5=9** **1+3+5+7=16**

1^2 2^2 3^2 4^2

Dot representation for triangular numbers:

<u>Triangular numbers:</u> are the numbers that can be represented by a triangle.



How many dots are added in each row?

Consecutive natural numbers are added each time.

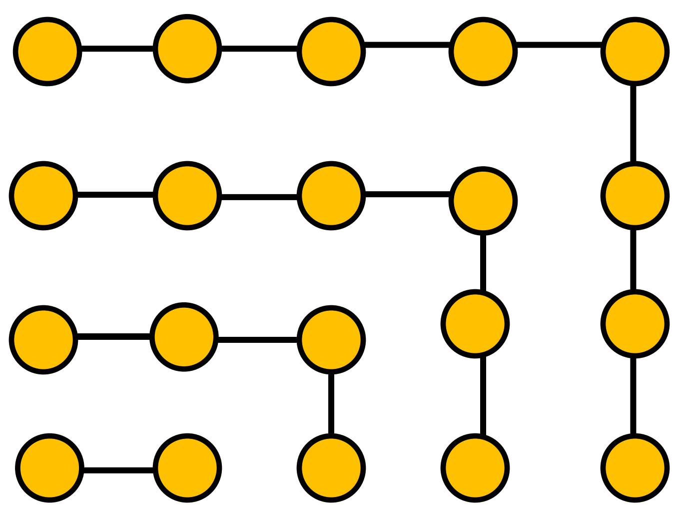
So the triangular numbers will be: 1, 3, 6, 10, 15, 21 ....

Dot representation for Oblong numbers:

Oblong numbers: numbers of the form n(n+1).

$1\times2, \quad 2\times3, \quad 3\times4, \quad 4\times5, \ldots\ldots\ldots\ldots\ldots$

The Oblong numbers will be: 2, 6, 12, 20 ………………

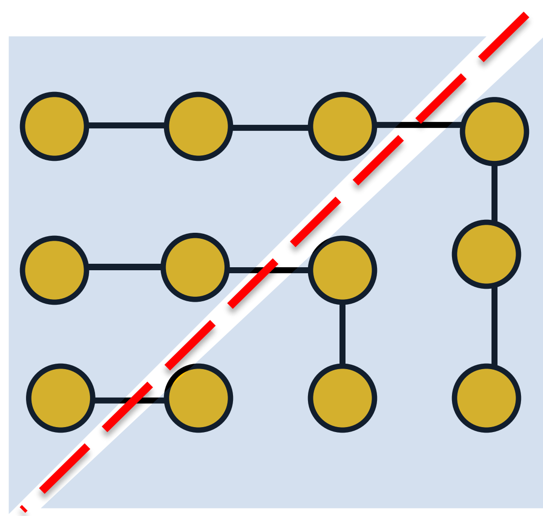


Any oblong number is the double of a triangular number:

The Oblong numbers are: 2, 6, 12, 20 ……………….

The triangular numbers are: 1, 3, 6, 10, 15, 21 ……………..

Oblong number 12



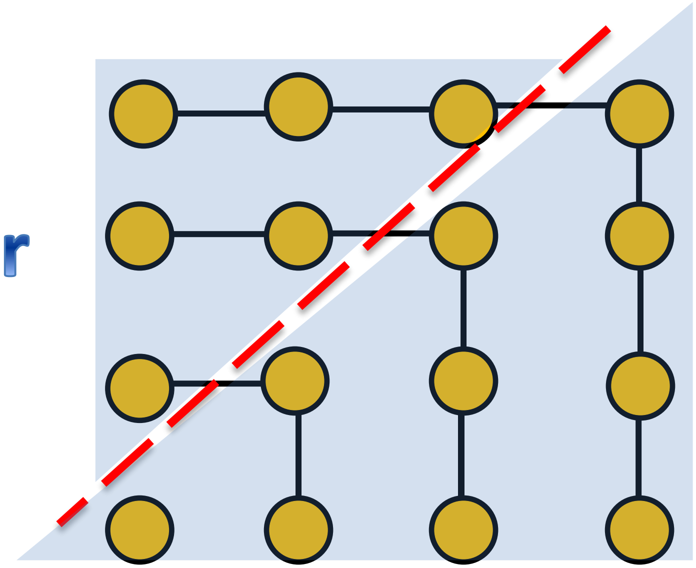
Triangular number 6 and 6

Any square number is the sum of two consecutive triangular numbers.

The square numbers are: 1, 4, 9, 16, 25 ……………..

The triangular numbers are: 1, 3, 6, 10, 15, 21 ……………..

Square number 16



Triangular numbers 6 and 10

Construction of Pythagorean Triples

Odd Number (n)

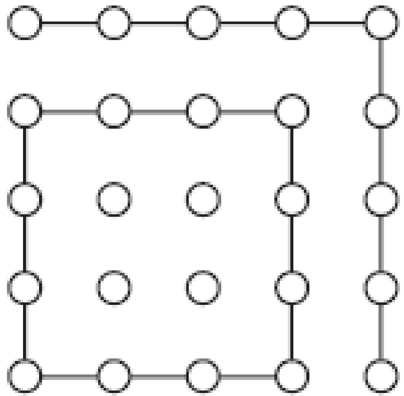
$$n, \frac{n^2 - 1}{2}, \frac{n^2 + 1}{2}$$

For Example:

n = 3

$$3, \frac{3^2 - 1}{2}, \frac{3^2 + 1}{2}$$

3 , 4 , 5



Any odd number is the difference of two consecutive squares

Construction of Pythagorean Triples

Even Number (m)

$$m\ ,(\frac{m}{2})^2-1\ ,(\frac{m}{2})^2+1$$

For Example:

m = 4

$$4\ ,(\frac{4}{2})^2-1\ ,(\frac{4}{2})^2+1$$

4 , 3 , 5

Construction of Pythagorean Triples

❖ **<u>Note:</u>** If the odd number is itself square, then the three square numbers have been found such that the sum of two numbers equals the third.

For Example:

n = 9

$$9\,,\frac{9^2-1}{2}\,,\frac{9^2+1}{2}$$

9 , 40 , 41

Let's check:

9 , 40 , 41

$$9^2 + 40^2 = 41^2$$

$$81 + 1600 = 1681$$

Note: We can easily prove the Pythagorean triple, if one of the terms is odd, then must be odd and one is even.

- **The geometric theorem of Pythagorean study is :**
- The square of any right triangle on the hypotenuse is equal to the sum of the squares on the legs.

- # There is no direct evidence of this.
- The theorem was known in other cultures long before Pythagoras lived.
- Nevertheless, it was the knowledge of this theorem by the fifth century B.C that led to the first discovery of what is today called an irrational number.

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History of mathematics
Group 3
P.39-41

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Section 4

Theorem " fifth century BCE"

↓

First discovery of irrational no.

Early Greeks

- Numbers connected with things counted.
- Counting requires that the units remain the same.
- The units themselves can never be divided or joined to other units.

Formal Greek Mathematics

↓

A number meant a **multitude composed of units** except number "1". So, Aristotle noted that the smallest number is "2".

Pythagoreans

- Every thing could be counted including lengths , so number one must have a measure.

- The measure was found in a particular problem, in which both side and diagonal of a square could be counted.

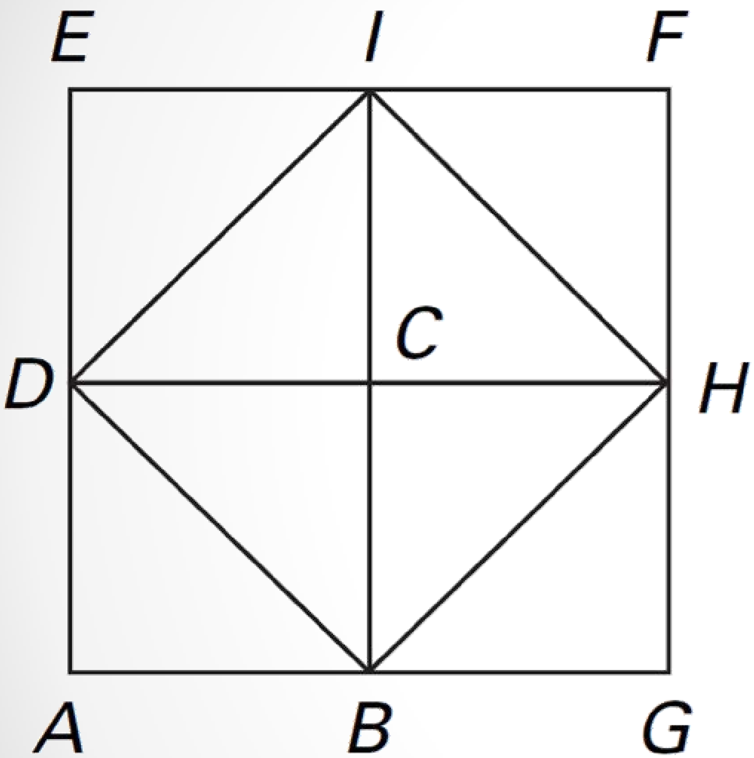
UNFORTNATLY ITS NOT TRUE.

- Because there is no common measure for diagonals and sides of a square.

- **What ever unit of measure is chosen such an exact number for the length the other line will need number+ portion of this unit and 1 cannot divide the unit.**

Cont'd

- **No one know who discover this result.**
- **scholar believes that the discovery took a place approximately 430 BCE.**
- **It became a crisis in Greek mathematics.**
- **this discovery simply opened up the possibility of new mathematical theories.**



Cause of the incommensurability and how discover it?

∵ *BD* and *DH* = commensurable

∴ each is represented by the number of times it is measured by their common measure.

∴One of them odd, would be a larger common measure.

DBHI and *AGFE* represent square numbers.

∵The *AGFE* square is double.

∴ it represents an even square number

side AG = DH = even number

square AGFE is a multiple of four

∵DBHI is half of AGFE

∴DBHI multiple of two

it represents an even square

∵Assumption: one of *BD, DH,* must be odd.

∴ two lines are incommensurable.

The notion of proof was ingrained into the Greek conception of mathematics. Although there is no evidence that the Greeks of the fifth century bce possessed the entire mechanism of an axiomatic system.

They certainly decided that some form of logical argument was necessary for determining the truth of a particular result. Furthermore, this entire notion of incommensurability represents a break from the **Babylonian** and **Egyptian** concepts of calculation with numbers.

Babylonians assign a numerical value to the length of the diagonal of a square of side one unit, but **Greek** was the first who form the notion that no "exact" value can be found. Although the Greeks could not "measure" the diagonal of a square, that line, as a geometric object was still significant.

Plato, in his dialogue Meno

Socrates: can you find a square whose area is double that of square of side two feet?

The boy suggests that each side should be doubled.

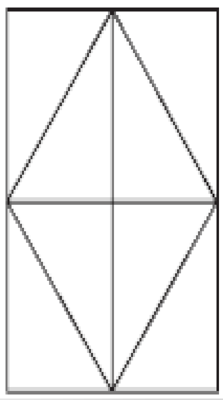
Socrates pointed out that this would give a square of area sixteen.

The boy's guess, that the new side should be three feet, is also evidently incorrect.

Socrates led him to figure out that if one draws a diagonal of the original square and then constructs a square on that diagonal, the new square is exactly double the old one.

But Socrates' proof of this is simply by a dissection argument

There is no mention of the length of this diagonal at all



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2.1.4 - Squaring the circle and doubling the cube

The idea of proof and the change from numerical calculation are further exemplified in the mid-fifth century

To solve two geometric problems

- **The duplication of the cube**

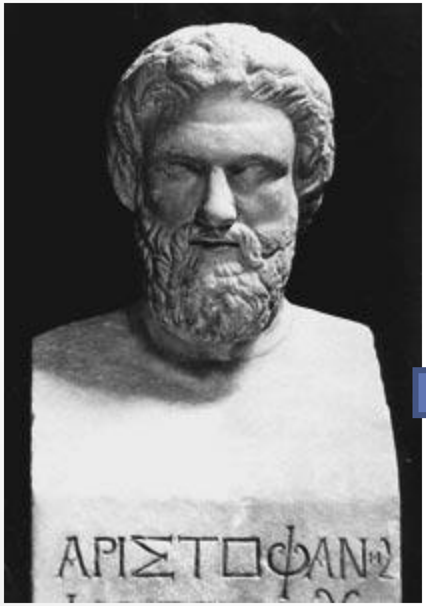
  (as noted in the oracle)

- **The squaring of the Circle**

  (already attempted in Egypt)

* The central goal of **Greek** mathematics was geometrical problem solving.
* The great body of theorems found in the major extant works of Greek mathematics served as logical underpinnings for these solutions.

Interestingly, these problem apparently could not be solved via the original tools of straightedge and compass was known to enough of the Greek public



Aristophanes

could refer to "squaring the circle" as something absurd in his play – the birds- .

Hippocrates of Chios was among the first to attack the cube and circle problems

The duplication of the cube: realized that the problem was analogous to the simpler problem of doubling a square of side a

that problem could be solved by constructing a mean proportional b between a and 2a

A length b such that a : b = 2a

For then $b^2 = 2a^2$

from the fragmentary records of Hippocrates work, it is evident that he was familiar with performing such construction.

Hippocrates of Chios

•**first to come up** with the idea of reducing the problem of doubling the cube of side a to the problem of finding two mean proportional
b, c between a and 2a.

for if $a:b = b:c = c:2a$

Then $a^3:b^3 = (a:b)^3 = (a:b)(b:c)(c:2a) = a:2a = 1:2$

And $b^3=2a^3$

He was not able to construct the two mean proportional's using the geometric tools at his disposal.

Hippocrates of Chios

The squaring of the Circle: essentially by showing that certain lunes (figures bounded by arcs of two circle) could be "squared", that their areas could be shown equal to certain regions bounded by straight lines.

http://www.gogeometry.com/geometry/lune_hippocrates_quadrature_index.html

Suppose that semicircle ABC is circumscribed about the isosceles right triangle ABC and that around the base AC an arc ADC of a circle is drawn so that segment ADC is similar to segments AB and BC; that is, the arcs of each are the same fraction of a circle, in this case, one-quarter (Fig. 2.11). It follows from the result on areas of circles that similar segments are also to one another as the squares on their chords. Therefore, segment ADC is equal to the sum of segments AB and BC. If we add to each of these areas the part of the triangle outside arc ADC, it follows that the lune $ABCD$ is equal to the triangle ABC.

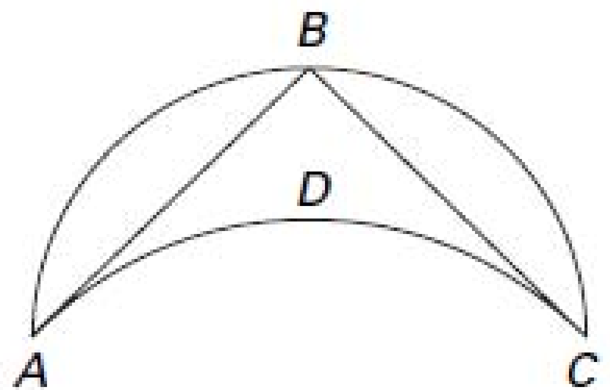


FIGURE 2.11

Hippocrates' lune on an isosceles right triangle

- Hippocrates gave constructions for squaring other lunes or combinations of lunes, he was unable to actually square a circle.

- it is apparent that his attempts on the squaring problem and the doubling problem were based on a large collection of geometric theorems .

- theorems that he organized into the first recorded book on the elements of geometry

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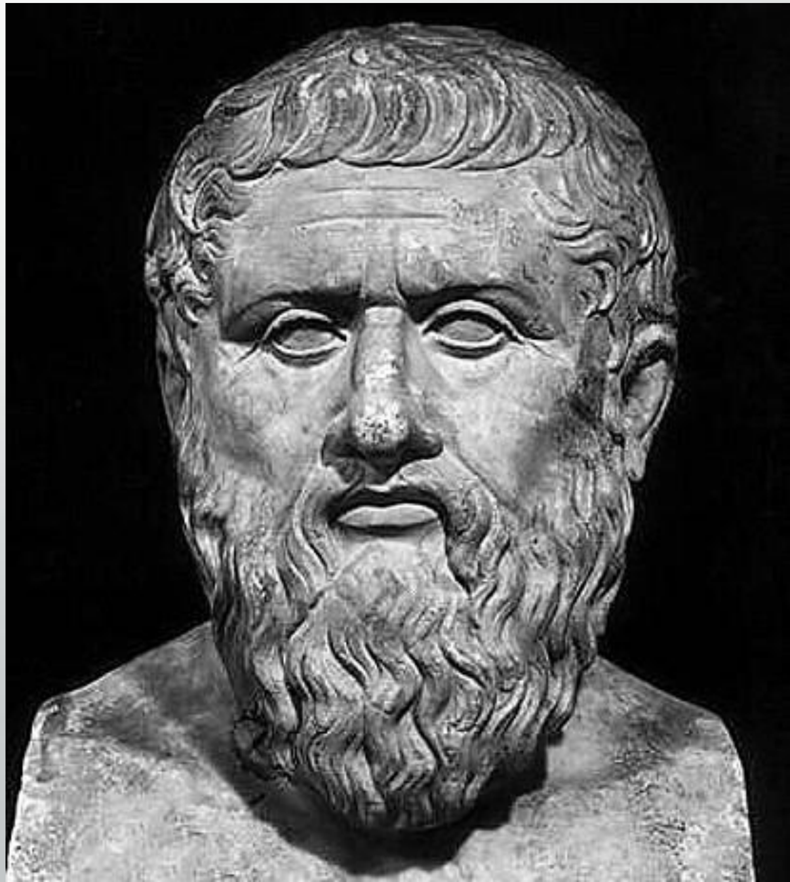
THE TIME OF PLATO (429-347 BCE)



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- Time of Plato saw significant effort made toward:

1. Solving problems (doubling cubes and squaring circles).
2. Dealing with incommensurability.
3. Its impact on the theory proportion.

- Scholars conducted seminars in math and philosophy with groups of advanced students.
- Over the entrance to the academy was inscribed

ΑΓΕΩΜΕΤΡΗΤΟΣ
ΜΗΔΕΙΣ ΕΙΣΙΤΩ

meaning "Let no one ignorant of geometry enter here"

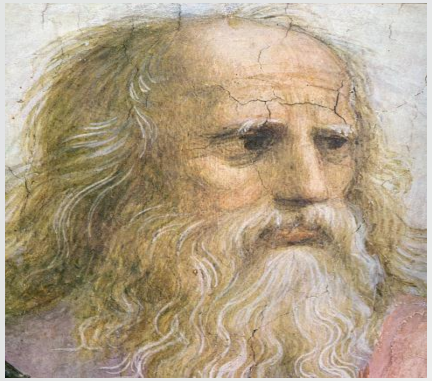
- The math syllabus inaugurated by Plato for students at the academy is described by him in his most famous work. Into two parts:

1. Education (philosopher- kings, the ideal rules of a state).
2. Mathematical (Arithmetic, plane geometry, solid geometry, astronomy, and harmonics).

- Mathematics is pursued for the sake on knowledge and not for commercial ends (studied for the training of the mind).
- What Plato writes includes the Pythagorean number theory and also additional materials that is included in Euclid's book.

- Limited amount of plane geometry is necessary for practical purposes.
- Mathematicians talk of operations in plane geometry الهندسة المستوية such as squaring or adding.
- According to Plato, is not to do something but to gain knowledge, “knowledge, moreover, of what eternally exists, not of anything that comes to be this or that at some time and ceases to be”
- In arithmetic, the study of geometry –and for Plato this means theoretical not practical, geometry—is for “ drawing the soul towards truth”
- It is important to mention that Plato …..

- The next subject of mathematical study should be solid geometry.
- Plato complained in the Republic that this subject has not been enough investigated.
- This because “no state thinks [it] worth encouraging” and because “students are not likely to make discoveries without director, who is hard to find.”
- Nevertheless, Plato felt that new discoveries would be made in this filed.
- In fact much done between dramatic date of the dialogue and time of Euclid.
- Some of which is included in books XI-XIII of the Elements.



Plato

Distinguished between the stars as material objects with motions , irregularities and variations and the ideal abstract relations of their paths and velocities expressed in numbers and perfect figures

Distinction is made in the final subject, of harmonics, between material sounds and their abstract.

Pythagorean

Discovered the harmonies that occur when strings are plucked together with lengths in the ratio of certain small positive integers.

Theory of ratio and proportion.

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THE BEGINNING OF MATHEMATICS IN GREECE ARISTOTLE

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CHAPTER 2

ARISTOTLE:

- **Aristotle (384-322 BCE): Studied at Plato's academy in Athens from the time he was 18 until Plato's death in 347.**
- **He wrote on many subject such as politics, ethics, physics and biology.**
- **His strongest influence was in the area of logic.**

logic:

- **Greeks were developing the notion of logic reasoning.**
- **He developed and posted the principles of logic argument.**

Aristotle believed that logical arguments should be built out of *Syllogisms*

Syllogisms : Consists of certain statements that are taken as true and certain other statements that are then necessarily true.

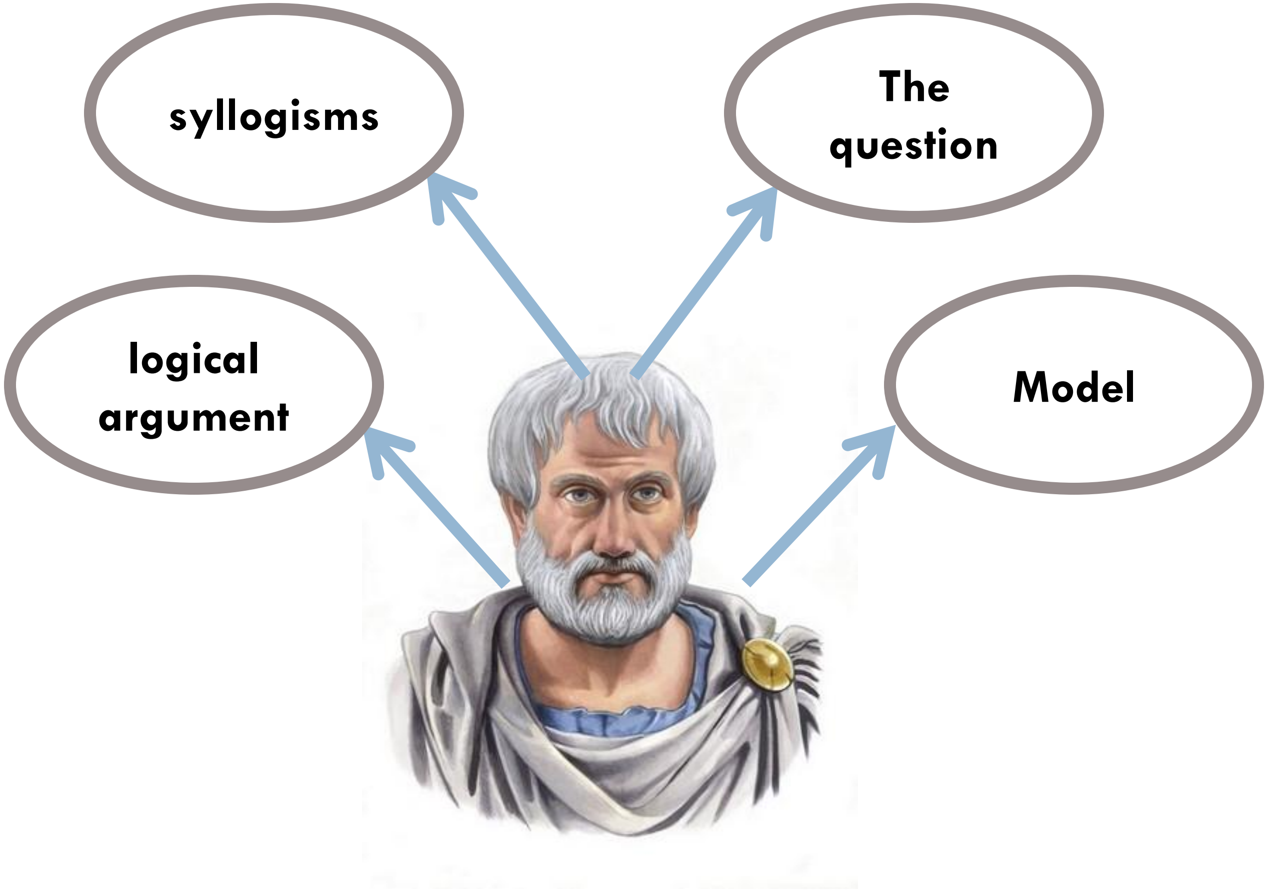
Aristotle distinguished between *Axiom* and *Postulates*

Axiom: One that are common to all. (Ex: Take equals from equals and equals remain)

Postulates : Basic truths that are peculiar to each particular science. (Ex: The definition of line and straight)

Aristotle basic principles of argument, principles that earlier thinkers had used intuitively.

First (true & false)
second (true or false)



Chrysippus basic rules of inference

Propositions are statements that can be either true or false.

Modus Ponens

If p, then q.

P.

Therefore, q.

Modus Tollens

If p, then q.

Not q.

Therefore, not p.

Hypothetical syllogism

If p, then q.

If q, then r.

Therefore, if p, then r.

Alternative syllogism

P or q.

Not p.

Therefore, q.

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The Beginnings of Mathematics in Greece

2.3.2 **Number versus Magnitude**
2.3.3 **Zeno's Paradoxes**

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TC2MA324 – History of Mathematics

Numbers versus Magnitude

One of the Aristotle contributions is the introduction into mathematics of the distinction between number and magnitude:

Discrete (number): indivisible unit.

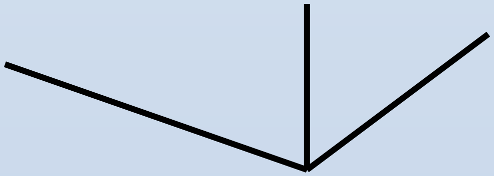
← **Quantity** →

Continuous (magnitude): divisible into divisibles that are infinitely divisible. For example: lines, surfaces, bodies and time.

Aristotle clarified this idea in his definition of "in succession" and "continuous"

Things are in succession if there is nothing of their own kind intermediate between them.
For example, the numbers 3 and 4 are in succession.

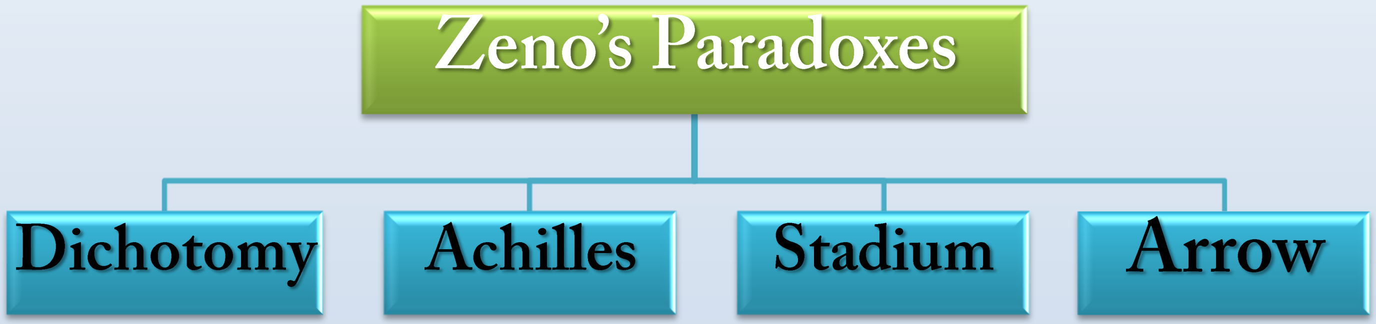
Things are continuous when they touch and when the touching limits of each become one and the same.
Line segments are therefore continuous if they share an endpoint.



Amani Abdulla

Aristotle (infinity, indivisibles, continuity, and discreteness)

To refute the famous paradoxes of Zeno

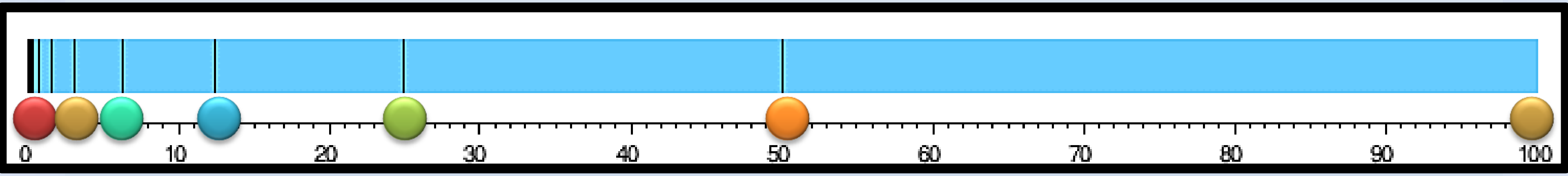


Fatima Sayed Yaseen

1st Paradox: Dichotomy

Non-existence of motion on the ground that that which is in locomotion must **arrive at the half-way stage** before it arrives at the goal

An object cannot cover a **finite distance** by moving during an **infinite sequence** of time intervals.



Zeno's Paradoxes

2nd Paradox

Achilles: "in a race, the quickest runner can never overtake the slowest, since the purser must first reach the point whence the pursued stated, so that the slower must always hold a lead."

Aristotle: ***time, like distance, is infinitely divisible.***

-But he is not bothered by an object covering an infinity of intervals in a finite amount of time.

-For "while a thing in a finite time cannot come in contact with things quantitatively infinite, it can come in contact with things infinite in respect to divisibility, for in the sense time itself is also infinite."

-In fact, given the motion in either of these paradoxes, one can calculate when one will reach the goal or when the fastest runner will overtake the slowest.

3rd Paradoxes

•show what happens when one asserts that a continuous magnitude is composed of indivisible elements.

•The Arrow states that "if everything when it occupies an equal space is at rest, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless." In other words, if there are such things as indivisible instants, the arrow cannot move during that instant. Since if, in addition, time is composed of nothing but instants, then the moving arrow is always at rest.

Aristotle: ***not only are there no such things as indivisible instants, but motion itself can only be defined in a period of time.***

•A modern refutation. on the other hand, would deny the first premise because motion is now defined by a limit argument.

Zainab Mahmood

Zeno's paradoxes

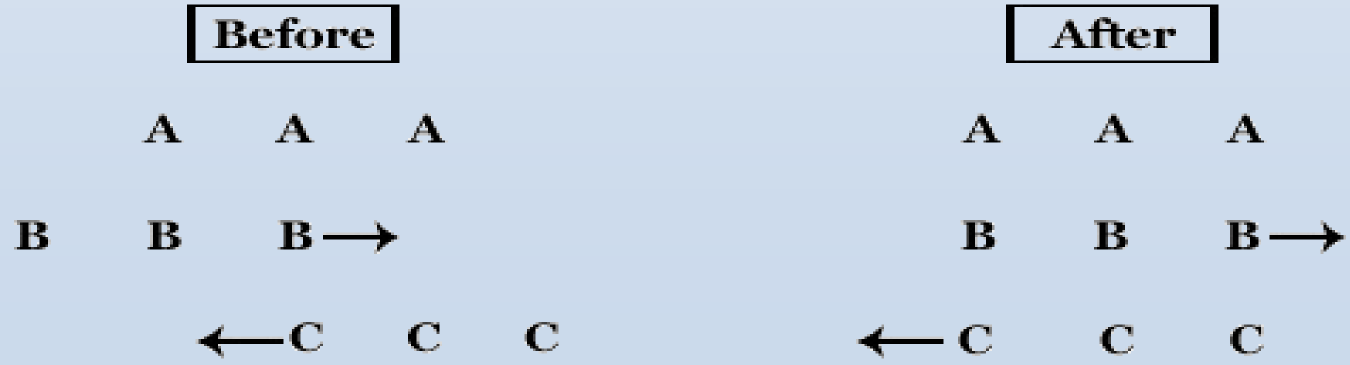
	space infinitely divisible	space infinitely indivisible
time infinitely divisible	Arrow	Achilles
time infinitely indivisible	Dichotomy	stadium

Aristotle refutes to Achilles and dichotomy by saying:

- Time, like distance, is infinitely divisible.
- In case of motion you cannot divide time or distance alone they must by divided together.

The paradox of the Stadium

- Three sets : A , B ,C
- A is still B moves to the right C moves to the left ..
- Indivisible space move in an indivisible unit of time
- No motion or indivisible instant.



Aristotle refutes this paradoxes by:

- Time is composed of divisible instants.
- Motion itself can only be defined in a period of time.
- Controversy regarding these paradoxes has lasted throughout history.
- The ideas contained in Zeno's statements and Aristotle's attempts at refutation have been extremely fruitful .
- Forcing mathematicians to the present day to think carefully about their assumptions in dealing with the concepts of the infinite or the infinitely small.

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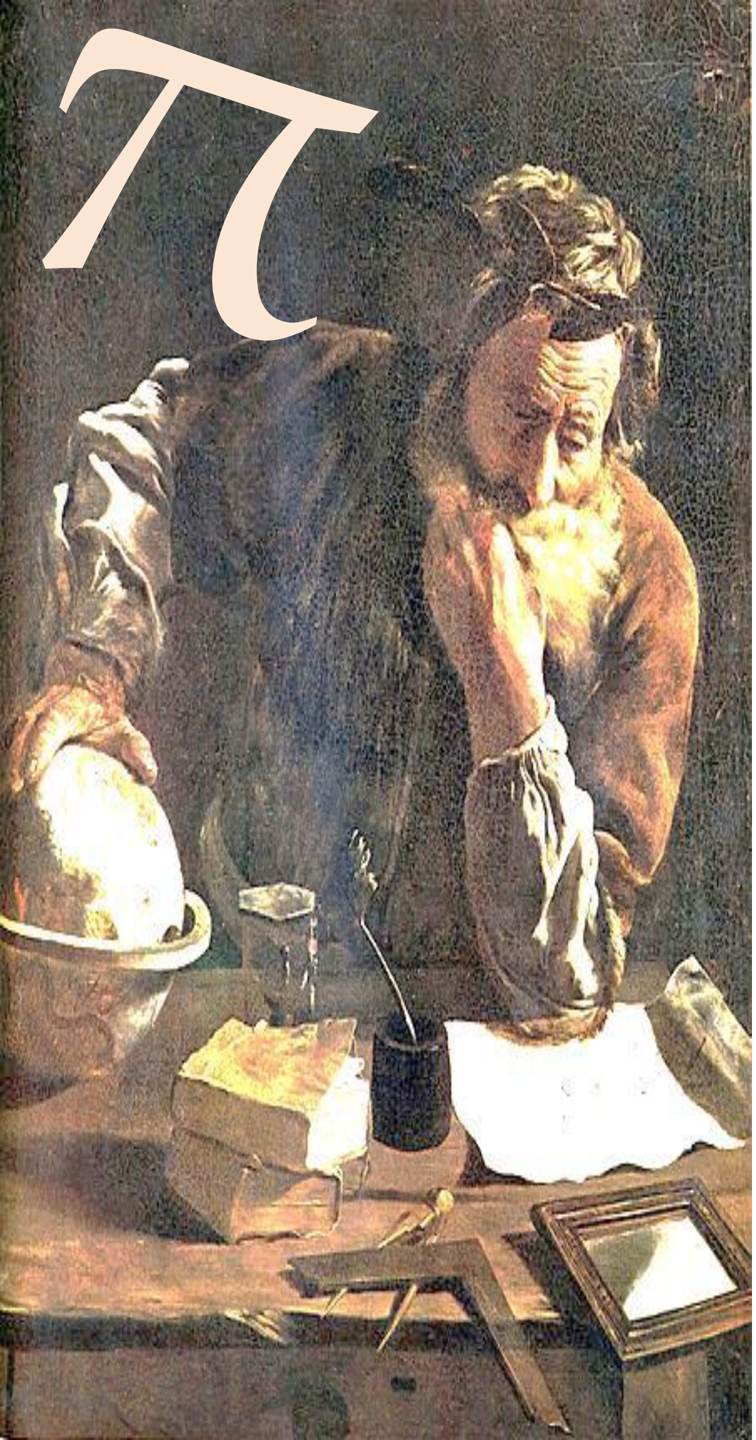

How Archimedes approximate pi (π)?

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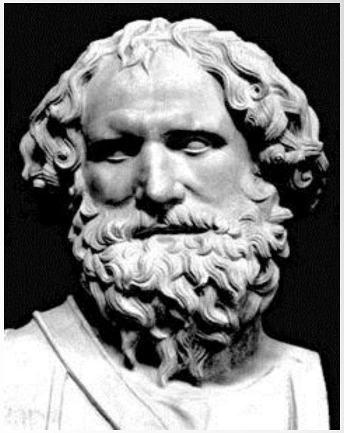
Archimedes (c.287 - c.212 BC)



- Archimedes was a Greek mathematician.
- philosopher and inventor who wrote important works on geometry, arithmetic and mechanics.
- Archimedes was born in Syracuse on the eastern coast of Sicily and educated in Alexandria in Egypt.
- He then returned to Syracuse, where he spent most of the rest of his life, devoting his time to research and experimentation in many fields.

pi (π)

π is one of the fundamental constants of mathematics: the ratio of a circle's circumference to its diameter. ($\pi = \frac{C}{D}$)

The Discoverer Scientist: The Greek mathematician Archimedes

How approximate pi (π)?

Why is Pi (actually) Important?

- Pi (π) is the **ratio** of the **circumference** of a circle to its **diameter**.
  - It **doesn't** matter how big or small the circle –
- π is **easy to observe**, but **hard to compute** accurately by hand.
- Methods to estimate π, demonstrate several significant advances in mathematics.

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