

Chapter 9: Social Choice

§ 1 - Two-candidates voting System

Suppose we have two candidates Alice (A) and Bob (B). We have odd number of voters. Each voter can choose either A or B.

Examples:-

- 1) Poll (Survey)
- 2) T/F
- 3) on/off
- 4) Quizzes.

voting System

- | | | |
|---|---|--|
| <p>① Majority rule
Who gets more votes, <u>wins</u>.</p> | <p>② Minority rule
Who gets <u>less</u> votes, wins.</p> | <p>③ Super-majority rule
Who gets above 70%, <u>wins</u>.</p> |
|---|---|--|

Properties:-

In any voting system with two candidates, we need the following:

- ① "Anonymity" Each voter is treated identically & unknown.
So if two voters exchange their ballot before submitting it, the result is unchange.
- ② "Neutrality" Each candidate is treated equally.
If Alice won and a new election and everyone voted Alice, will vote for Bob, then Bob wins.
- ③ Monotonicity:
If a new election, one who voted for Bob, change to Alice, the Alice still wins.
- ④ Decisiveness: There will be a Unique winner.

May's Theorem

Majority rule is the only voting system that satisfies (1)-(4).

Proof: (See Research Project).

§2 - Three or more Candidates

Example:

	Reem (3)	Aysha (4)	M	C	M (2)	C (2)	I
1 st	M	I	M	C	M	C	I
2 nd	C	C	A	M	I	A	M
3 rd	A	M	C	I	C	M	C
4 th	I	A	I	A	A	I	A

① Condorcet method

A candidate is a winner if s/he would defeat in one-to-one competition using the majority rule every other candidate.

So X is the winner, if for every other candidate Y, more than half of the voters rank X above Y.

Exercises:

①						②	5	7	3
1 st	C	B	B	A	C	2	A	C	B
2 nd	B	A	C	B	A	1	B	B	C
3 rd	A	C	A	C	B	0	C	A	A

A=4, B=6, C=5